University of Nottingham
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Linkage Mechanisms

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Adapted from the work of Dr Hengan Ou

Adapted from **R.L. Norton, "Design of Machinery, an introduction to the synthesis and analysis of mechanisms and machines",** McGraw-Hill, 3rd edition, 2004

Outlines

- Introduction to Mechanisms and Linkages
- Types of Rigid Body Motions
- Kinematic Pairs
- Kinematic Representation of Mechanisms
- Degrees of Freedom (Mobility)
- Mechanisms for Different Engineering Applications

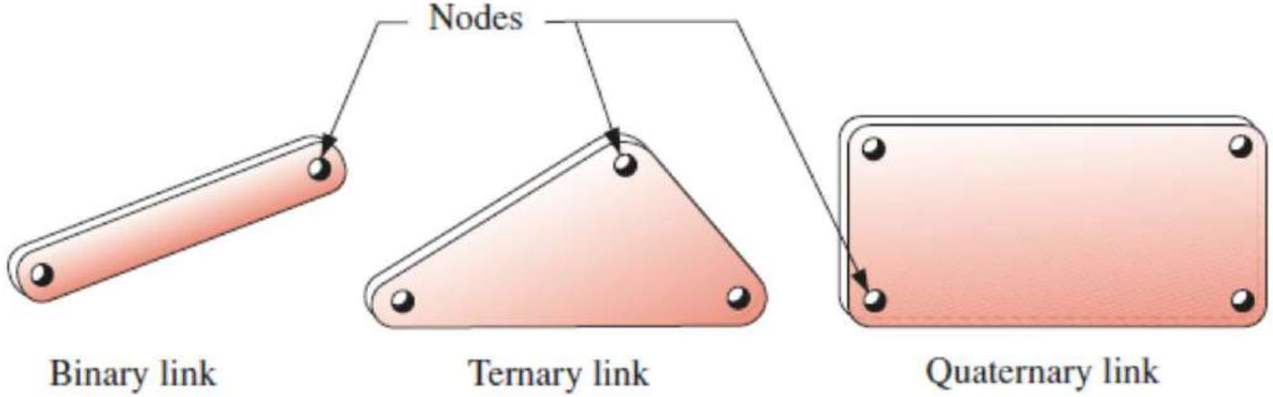
Learning Objectives

- To be familiar with types of motion, links, joints and kinematic chains
- To be able to determine Degrees of Freedom (DoF) of a common linkage mechanism
- To be familiar with common types of linkage mechanisms

** This session should be complementary to the theories/methods you learnt in evaluating position, velocity and acceleration or dynamic forces as part of **MMME2046**.

Links, joints and kinematic chains

- **Linkages** are the basic building blocks of all mechanisms and they are made of **links** and **joints**.
- A **link** is a rigid body that has at least two nodes (points for attachment to other links)
  - Binary link: with TWO nodes
  - Ternary link: with THREE nodes
  - Quaternary link: with FOUR nodes

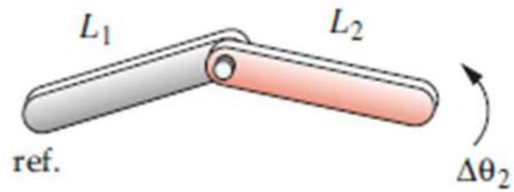


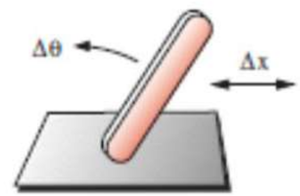
Links, joints and kinematic chains

- **A joint** (or kinematic pairs) is a connection between TWO or more links (at their nodes) to allow relative motion between connected links.

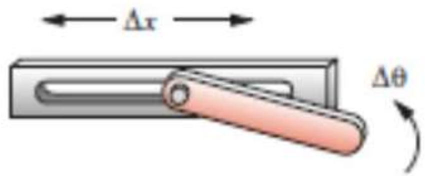
- **Joints** may be classified by the following ways:
  - By the type of contact between links, e.g. line, point or surface
  - **By the number of DoF allowed at the joint**
  - By the type of physical closure, e.g. force or form closed
  - By the number of links joined (order of the joint)

- **Lower pair** or **higher pair** of joints
  - A **lower pair** is a joint with surface contact (pin in a hole)
  - A **higher pair** is a joint with point or line contact





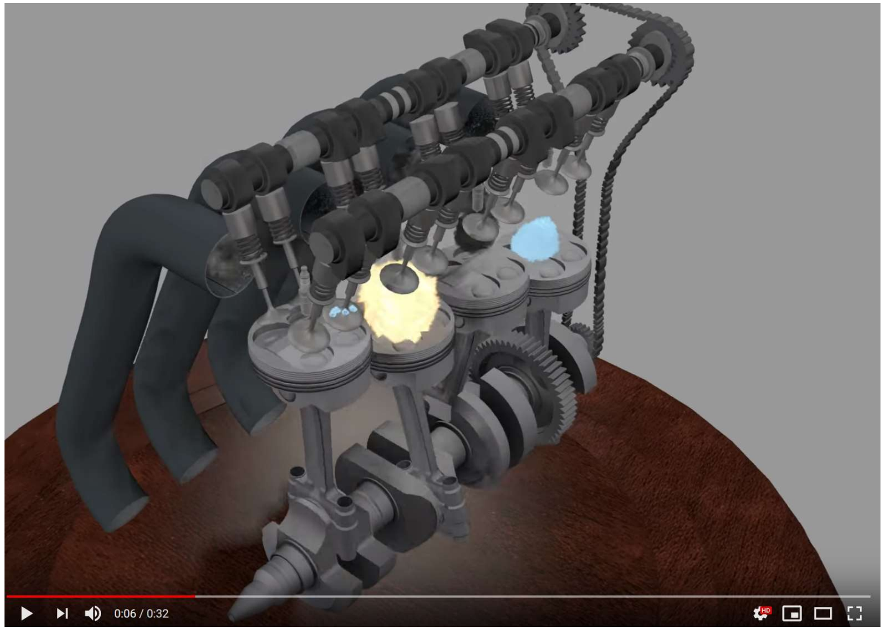
Link against plane (force closed)



Pin in slot (form closed)

Why linkage mechanisms?

- **Linkage mechanism** is a machine system involving **links, joints** and **linkage chains** that enable transformation of **motion, force and power** for specific functions.



https://www.youtube.com/watch?v=5TxrR17snKE

https://www.youtube.com/watch?v=Klz50qzM81w
https://en.wikipedia.org/wiki/Excavator#/media/File:Excavator_mechanism.gif

Types of Motion: Freedom & Constraints

- The motion of any rigid body can be considered to be a **combination of translation and rotational motions**

- **Translation motion** is a movement which can be resolved into components along one or more of the three axis x,y or z

- **Rotational motion** is one which has components rotating about one or more of the axis

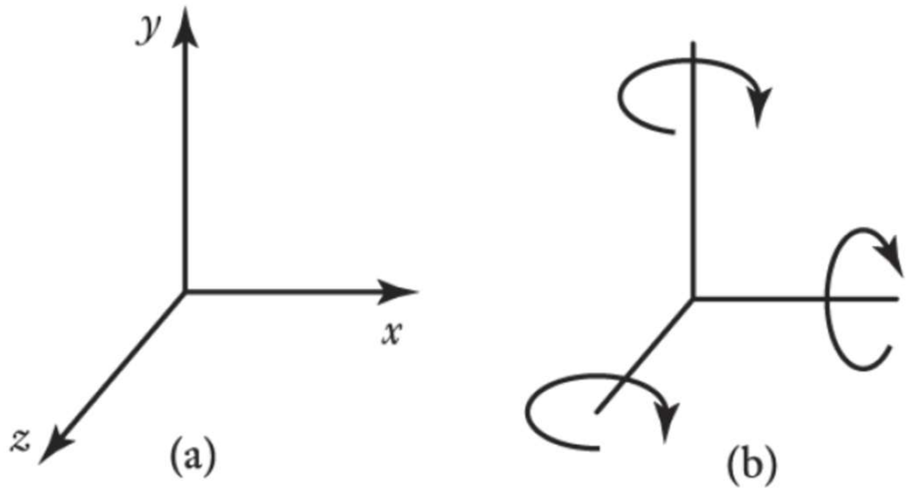


Figure 8.1 Types of motion: (a) translational, (b) rotational

Types of Motion

A complex motion is the combination of the translation and rotation motions

Example of complex motion:

- Instruct a robot to pickup a pencil from a table: you should breakup motion into small simple motion, e.g. instruct joint 1 to rotate by 20 degree then link 2 to be extended by 4 mm….etc

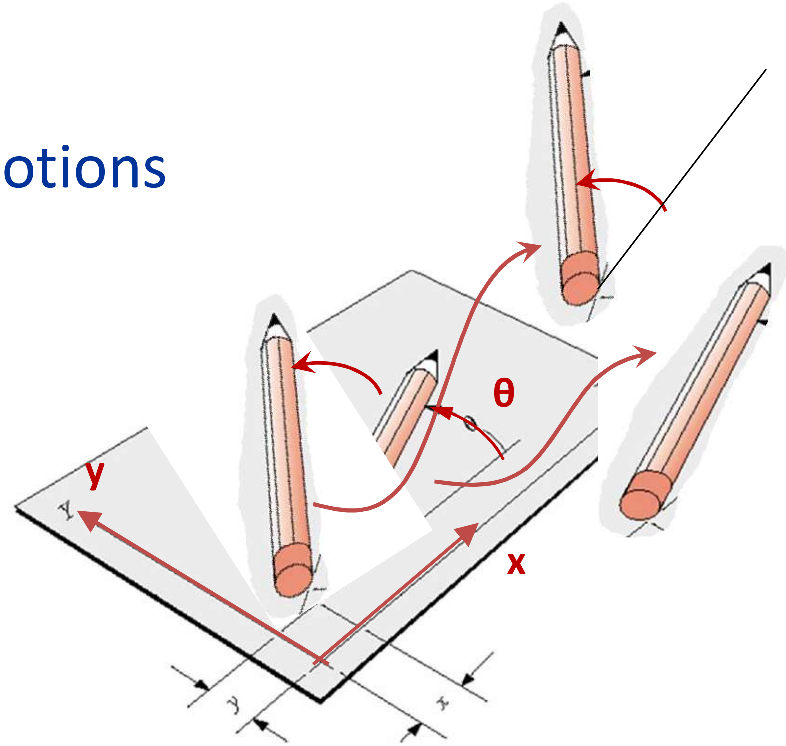
Types of motion

- **Pure translation:** all points of the pencil move in a curvilinear path without change of angular orientation
- **Pure rotation:** the pencil rotates around its rubber end to enable only a change of its angular orientation
- **Combination of both (complex motion):**

  a combination of rotational & translation motions simultaneously

- **Degrees of Freedom (DoF):** the number of independent parameters needed to uniquely define its position of a rigid body (the pencil)

  e.g. the pencil has **3 DoF** (**x, y, θ**)



Types of Motion:

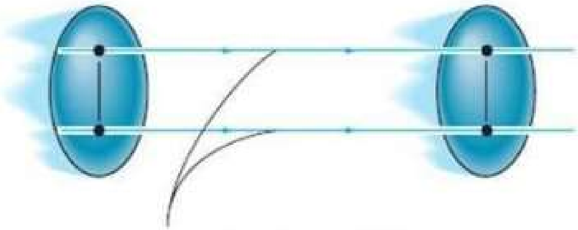
- **Rectilinear translation:** points in the body move in parallel straight lines.

- **Curvilinear translation:** points in the body move along identical curves, and so the link does not rotate with respect to the ground.

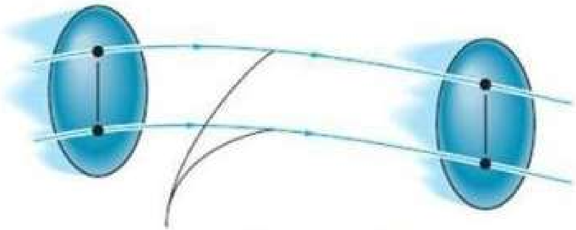
- **Rotation:** points in the body rotate about a single point, which is usually fixed to the ground.

- **General planar motion:** a general combination of rotation and translation.

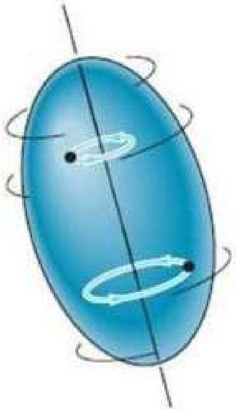
Types of Rigid Body Motion



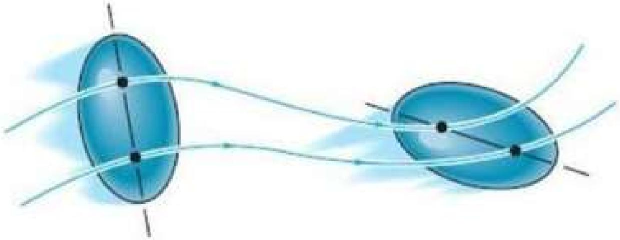
Path of rectilinear translation



Path of curvilinear translation

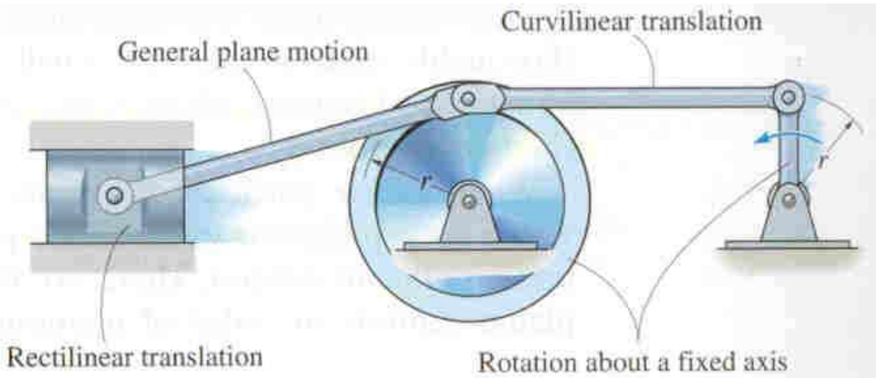


Rotation about a fixed axis

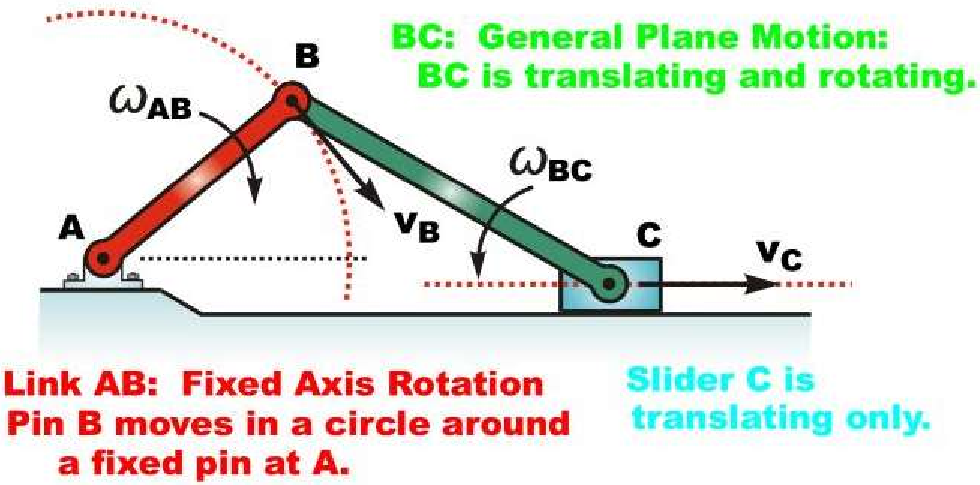


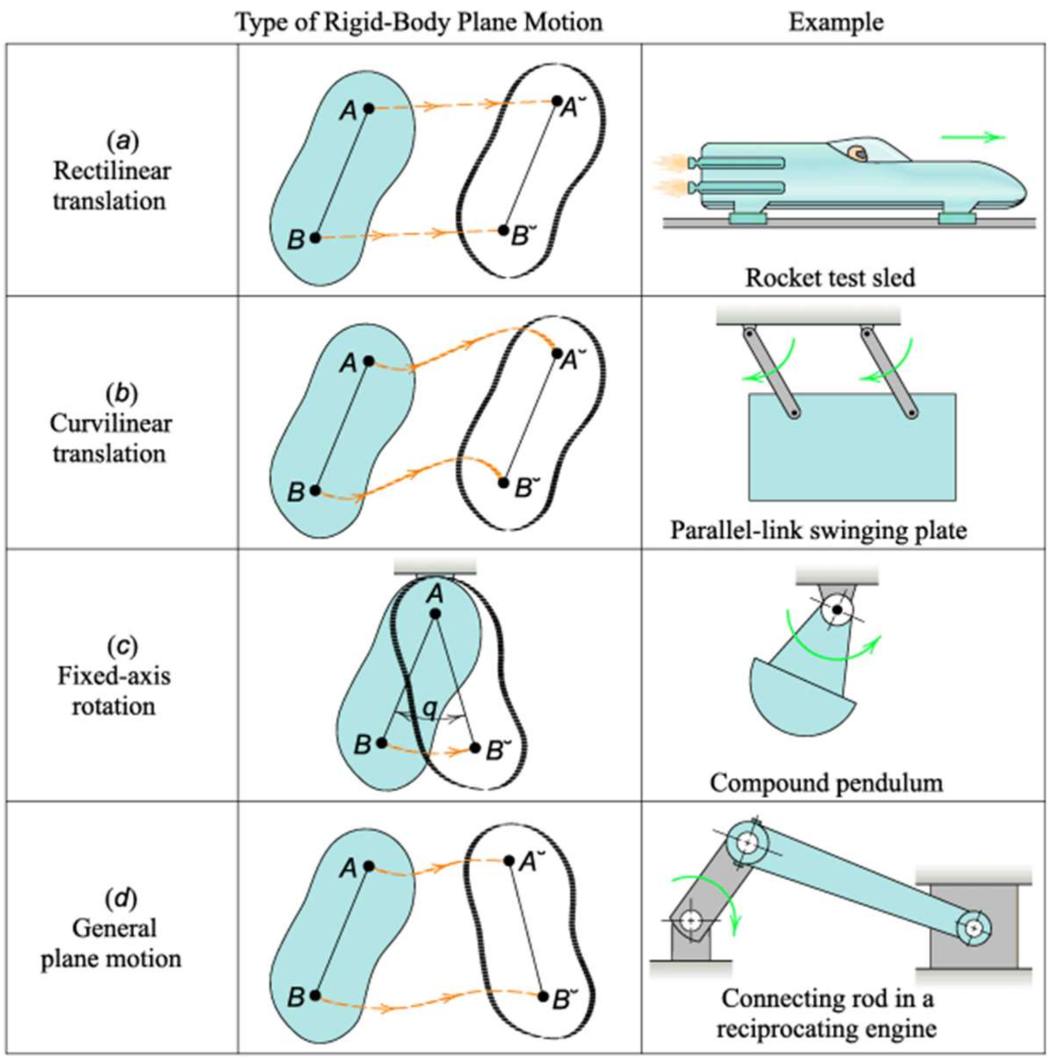
General plane motion

Types of Rigid Body Motion



Example: "Slider Crank" Mechanism
Illustrates all three motions...

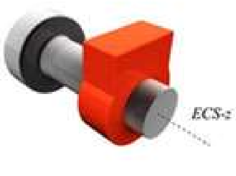




Joints (Kinematic Pairs)

Joint Type	Allowable Relative DOFs
Join	-
Spherical	θ_x , θ_y , θ_z
Cylindrical	w , θ_z
Slot	u , θ_x , θ_y , θ_z
Revolute	θ_z
Planar	u , v , θ_z
Translational	u
Universal	θ_x , θ_z
General	User defined

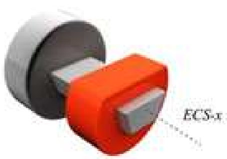
Spherical



Cylindrical

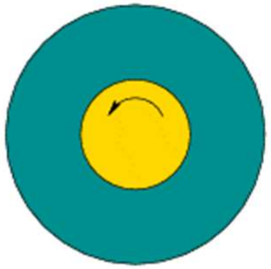
Revolute

Planar

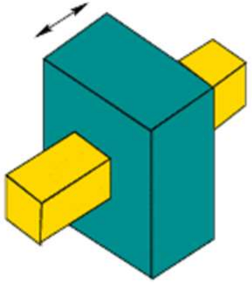


Translational

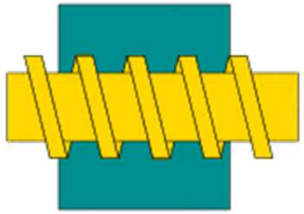
General



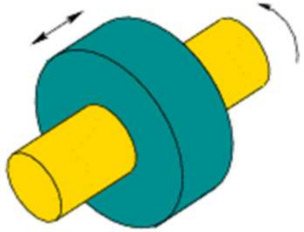
Revolute
1 Degree of Freedom



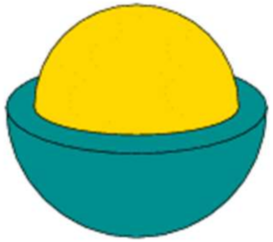
Prismatic
1 Degree of Freedom



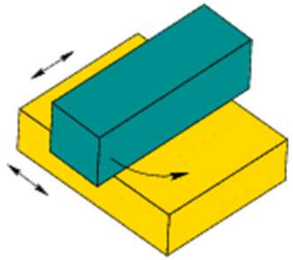
Screw
1 Degree of Freedom



Cylindrical
2 Degrees of Freedom



Spherical
3 Degrees of Freedom



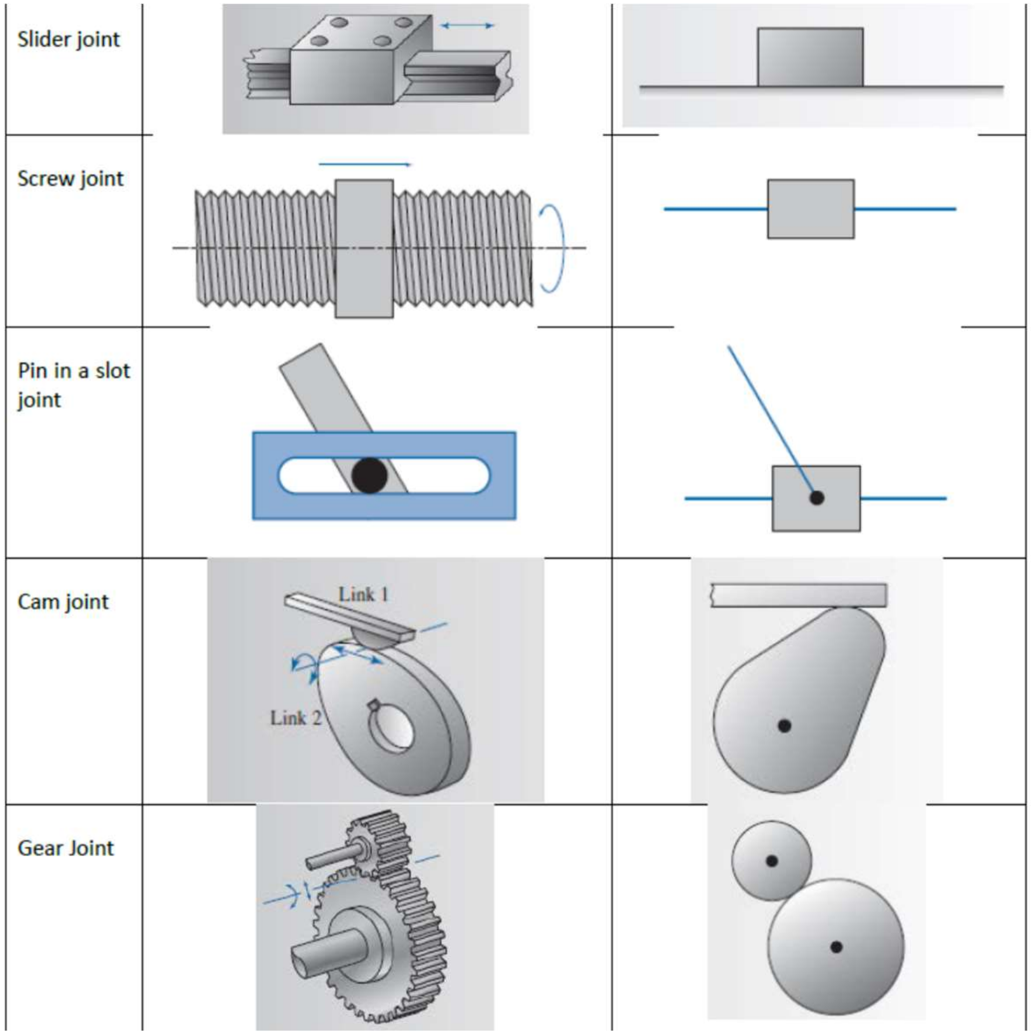
Planar
3 Degrees of Freedom

Kinematic Representation of Mechanisms

A kinematic diagram should be drawn to a scale proportional to the actual mechanism.

For convenient reference, the **links are numbered**, starting with the frame, which serves as the frame of reference for the motion of all other parts.

To avoid confusion, the **joints should be lettered.**



Introduction to Mechanisms

What is a mechanism?

A set of rigid bodies, connected so as to move with definite relative motion

Mechanism sub-types:

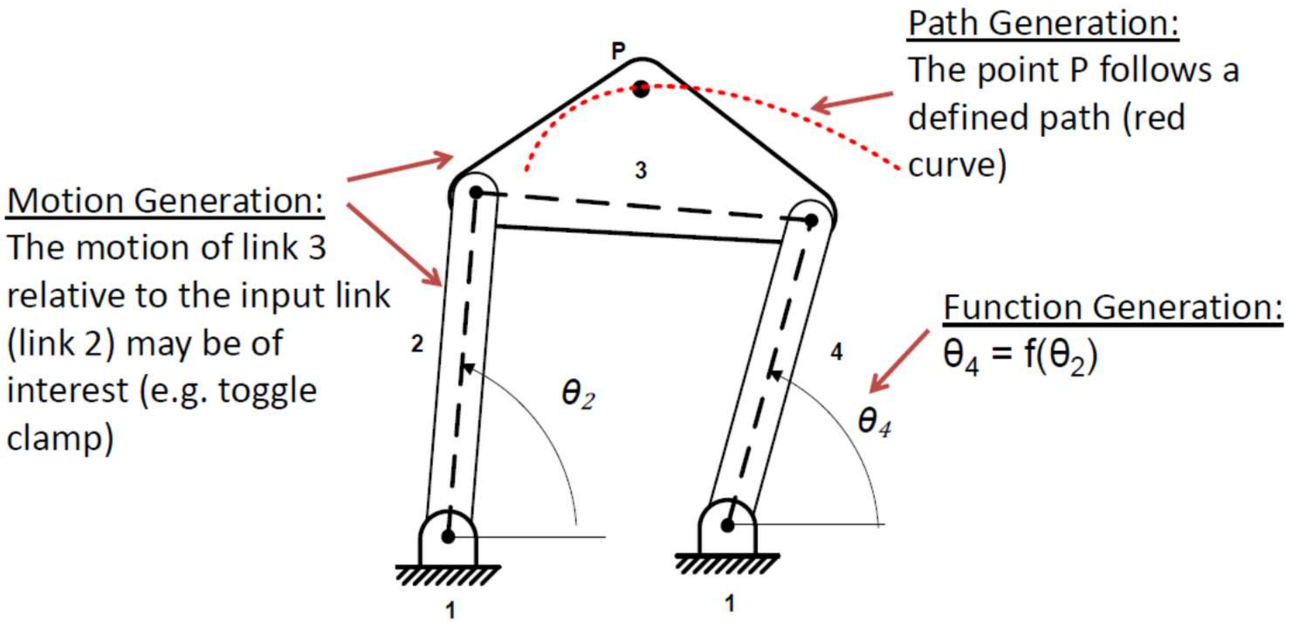
- **Planar**: All bodies move in parallel planes (and all forces act in parallel planes)
- **Spherical**: All bodies move on a sphere, and rotate about normals to the sphere
- **Spatial**: Up to 3 translational and 3 rotational Degrees Of Freedom (DOF) are possible

Tasks of Mechanisms

Knowledge of the kinematic & dynamic performance of mechanisms is critical to the design of mechanical components

Tasks of mechanisms can be classified in three groups:

- Function generation
- Path generation
- Motion generation

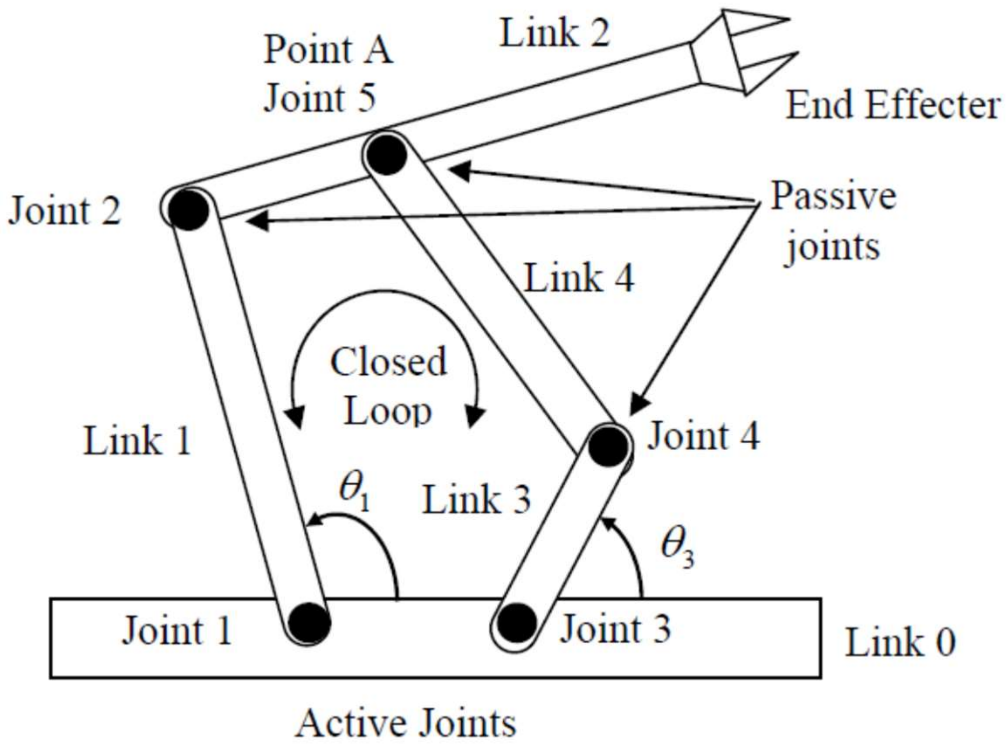


Parallel Linkages

It should be noted that, if more than two joints were actively driven by independent actuators, a conflict among three actuators would occur due to the closed-loop kinematic chain.

Three of the five joints should be ***passive joints***, which are free to rotate.

Only two joints should be ***active joints,*** driven by independent actuators.

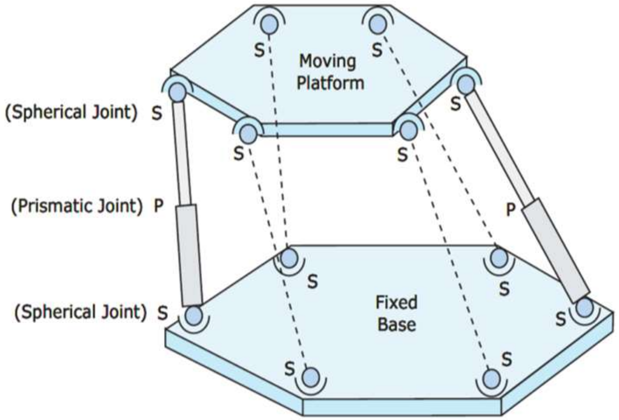


Parallel Linkages

Stewart mechanism consists of a moving platform, a fixed base, and six powered cylinders connecting the moving platform to the base frame.

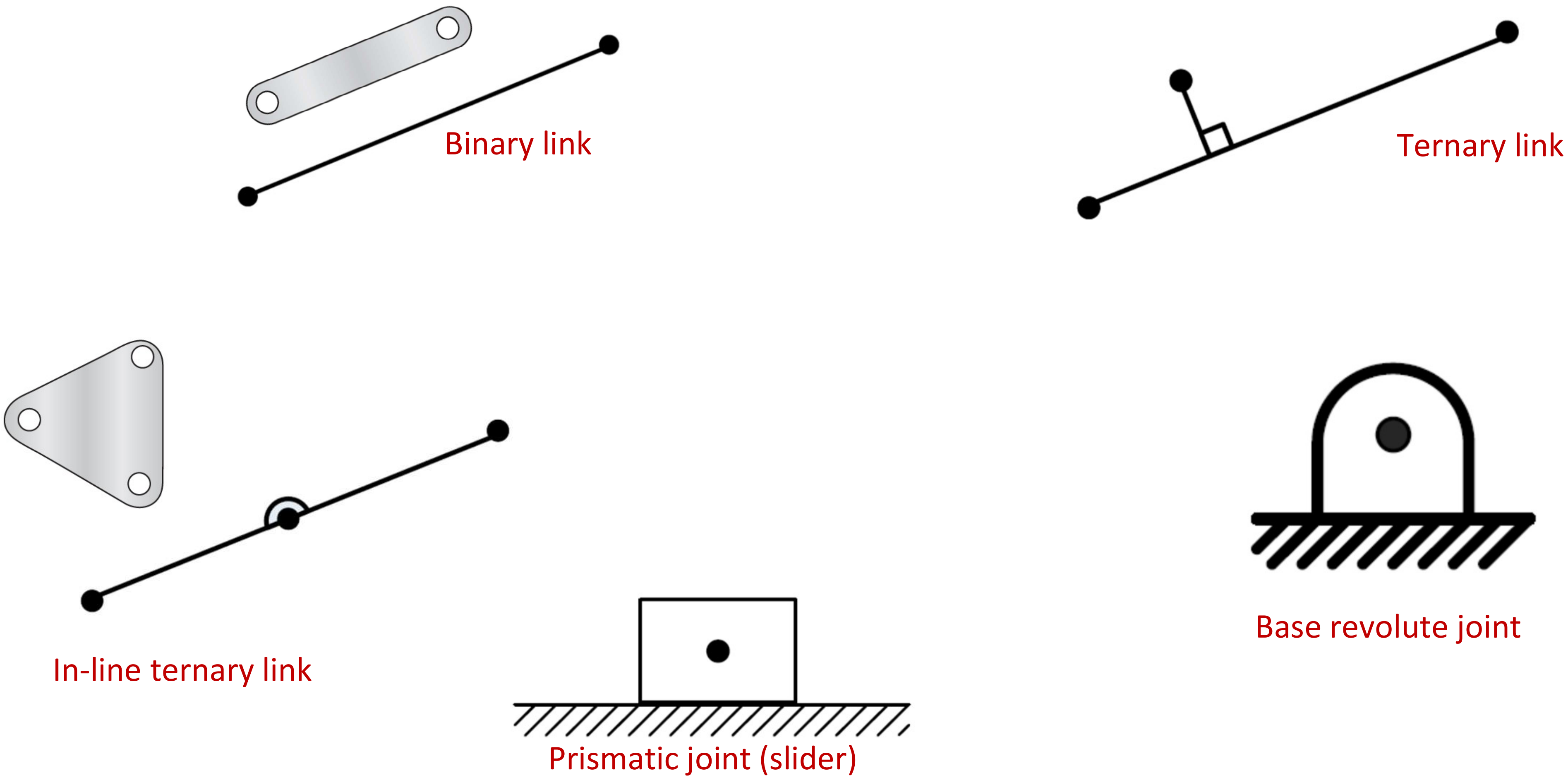
The position and orientation of the moving platform are determined by the six independent actuators. The load acting on the moving platform is born by the six "arms".

Therefore, the load capacity is generally large, and dynamic response is fast for this type of robot mechanisms.

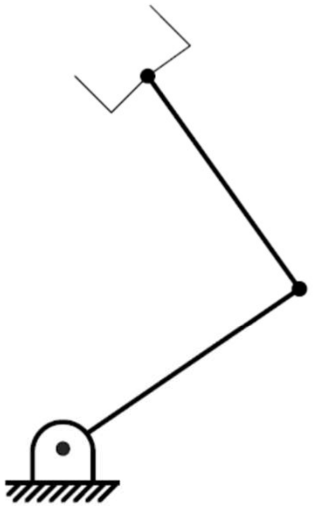


Stewart parallel mechanism

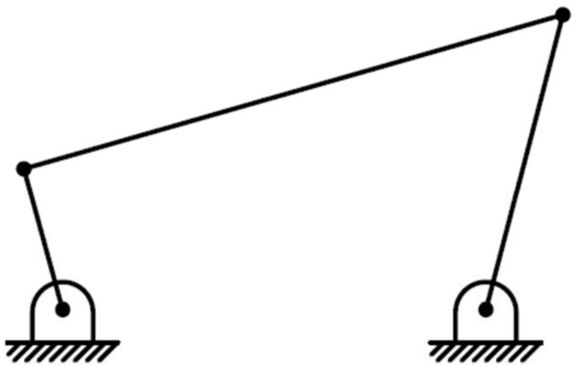
Kinematic Representation of Mechanisms



Kinematic Representation of Mechanisms



Open chain:
Planar manipulator

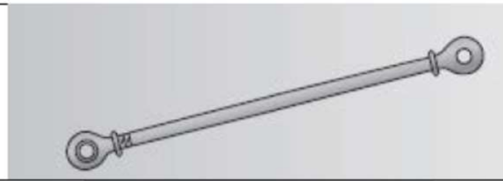
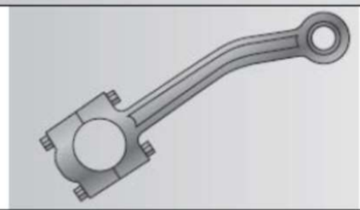
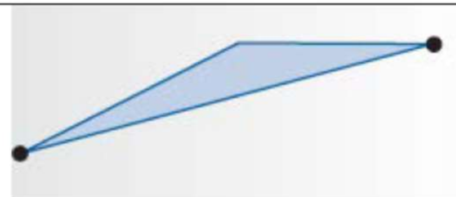
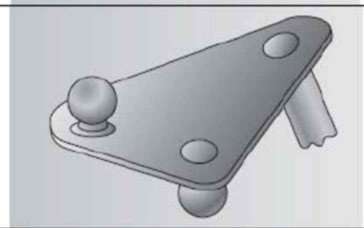
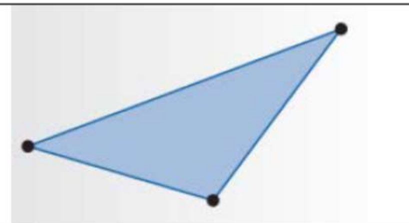
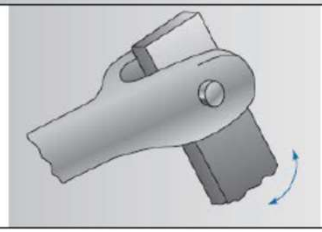
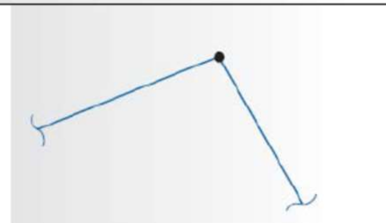


Closed chain:
4-Bar mechanism

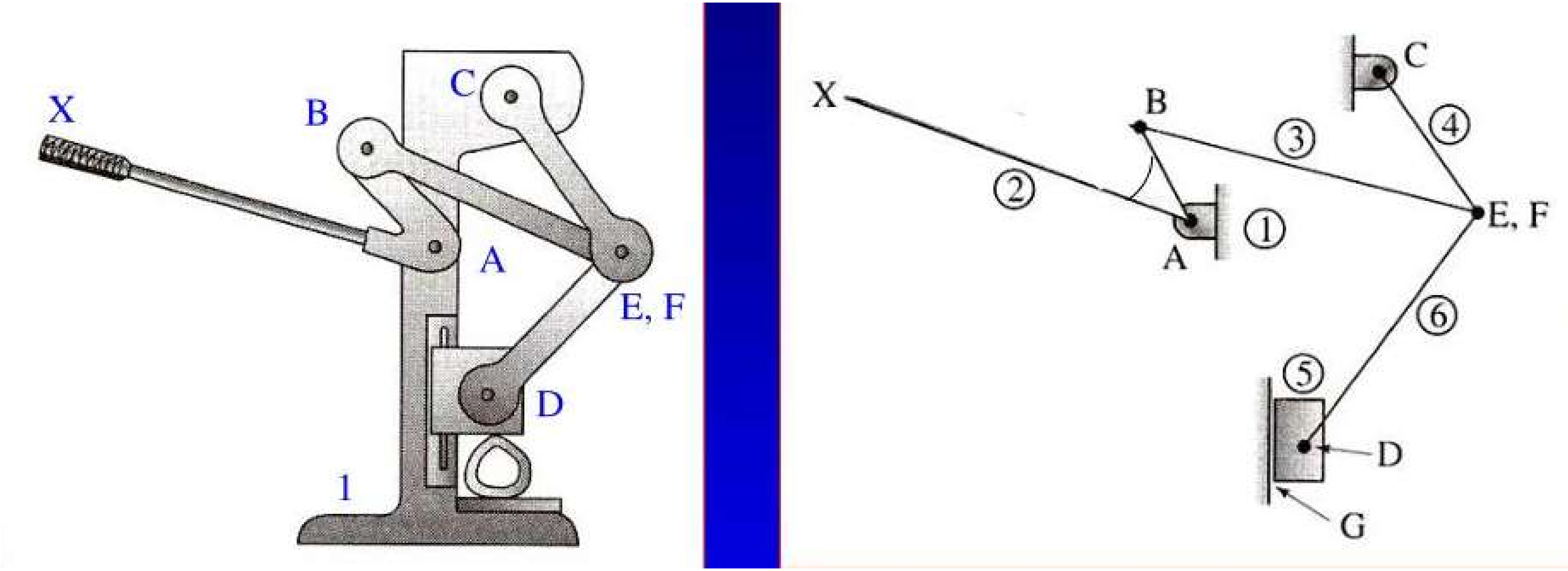
Kinematic Representation of Mechanisms

Mechanisms are depicted schematically using kinematically equivalent diagrams (skeleton sketches) to simplify visualization and analysis

In analyzing the motion only the dimensions that affect the motion are shown.

Component	Typical form	Kinematic representation
Simple link		
Simple link with point of interest		
Complex link		
Pin joint		

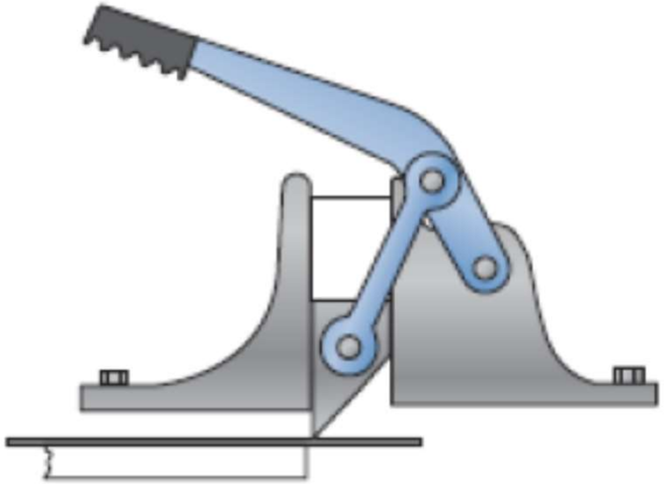
Kinematic Representation of Mechanisms



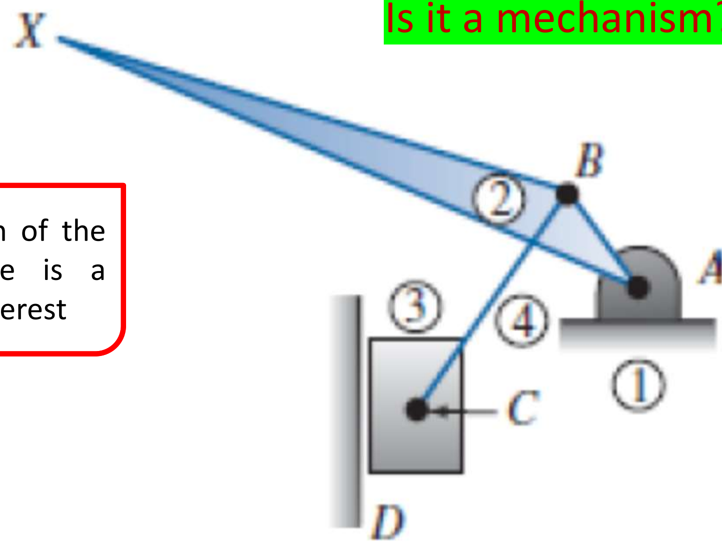
Can you calculate the DOFs?
Is it a mechanism? Why?

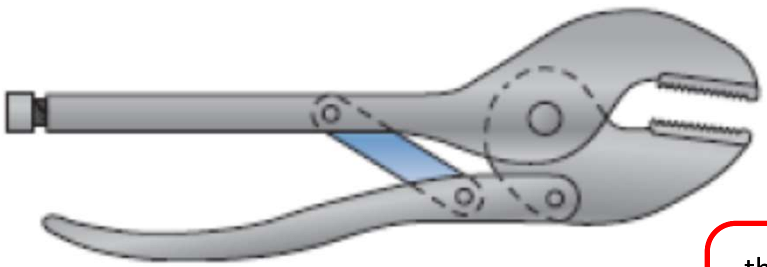
Kinematic Representation of Mechanisms

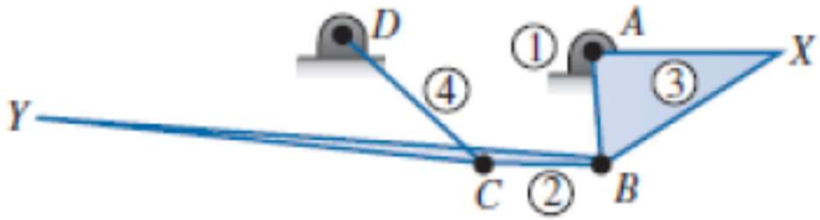
Can you calculate the DOFs?
Is it a mechanism? Why?



the motion of the bar handle is a point of interest







the motion of the end of the bottom jaw and the lower handle are points of interest.

Kinematic Chains

Examples of links: levers, cranks, connecting rods & pistons sliders, pulleys, belts and shafts

A sequence of joints and links is known as kinematics chain Fig 8.5a

Consider motor car engine where the reciprocating motion of a piston is transformed into rotational motion of a crankshaft on bearings mounted in a fixed frame.:

- Link 1 crankshaft
- Link 2 the connecting rod
- Link 3 the fixed frame,
- Link4 the slider i.e. the piston which move relative to the fixed frame

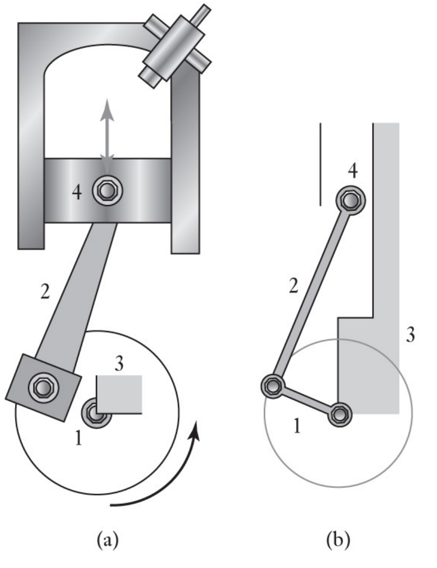


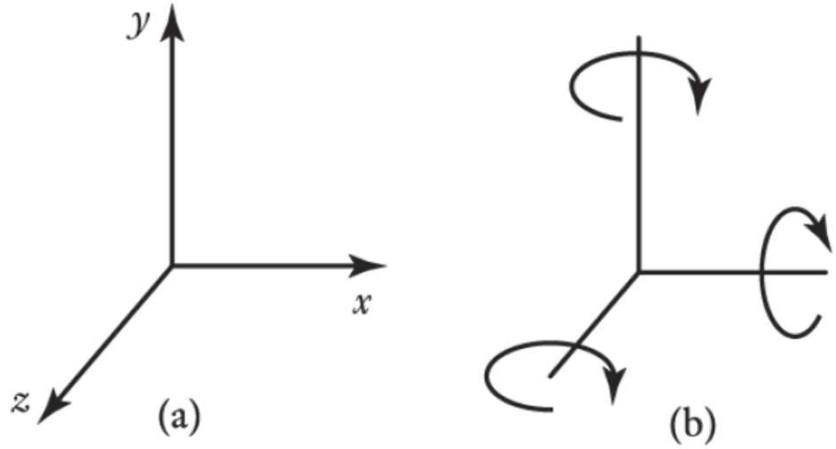
Figure Simple engine mechanism

Freedom and constraints

- A body that is **free in space** can move in three independent, mutually perpendicular directions and rotate in three ways about those directions.

- It is said to have **six degree of freedom (6 DOFs).**

- The number of degrees of freedom are the number of components of motion that are required in order to generate the motion.



Freedom and constraints

- If a joint is **constrained** to move along a line (one single coordinate axis) then it has one degree of freedom (1 DOF)

- If a joint is constraint to move on a plane (2 dimensions; xy plane) then it has two degree of freedom (2 DOFs)

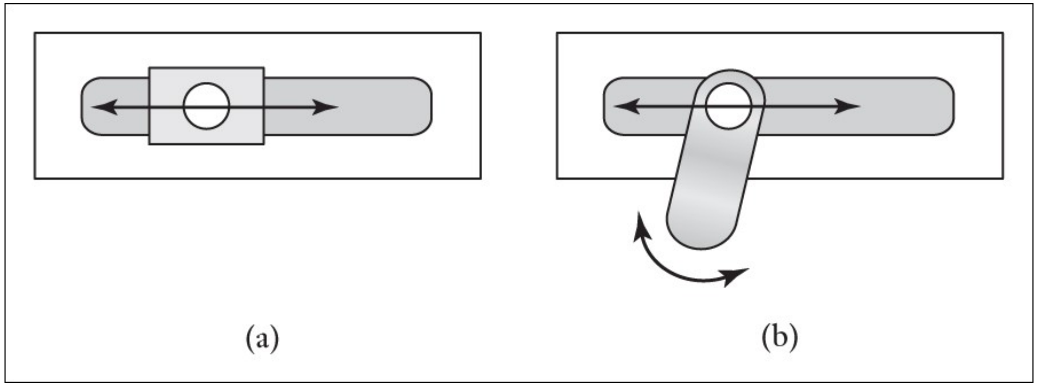


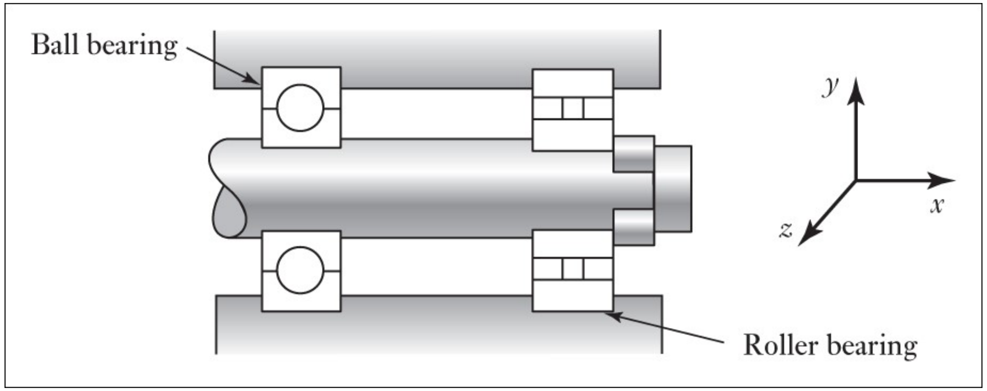
Figure Joints with: (a) one, (b) two degrees of freedom.

Freedom and constraints

- The problem in a design is to reduce the number of DOF which requires an appropriate number and orientation of constraint.

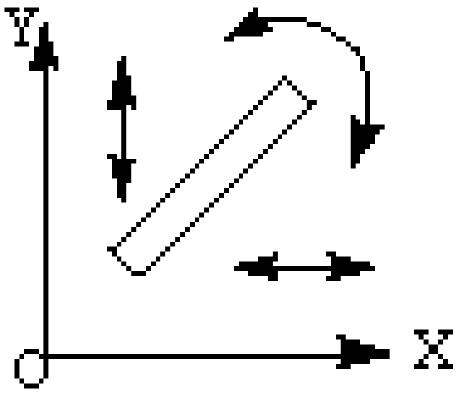
- **Fixed body implies zero DOF** and implies 6 constraints.

- Concept in design:
  In fixing a body or guiding it to a particular type of motion, the minimum number of constraints should be used "kinematics design".

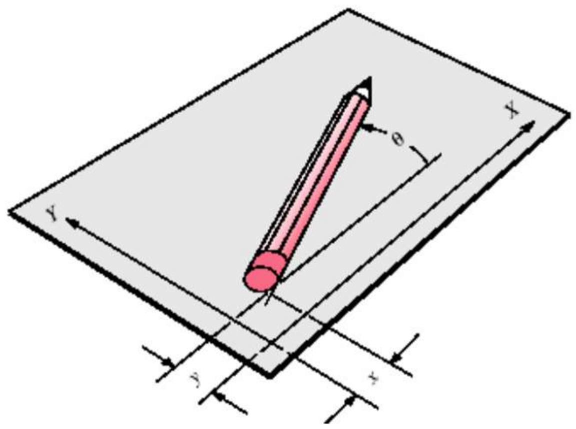


Degrees of Freedom (DOFs)

The degrees of freedom **(dof)** of a system are the number of independent coordinates which are required to uniquely define its position.

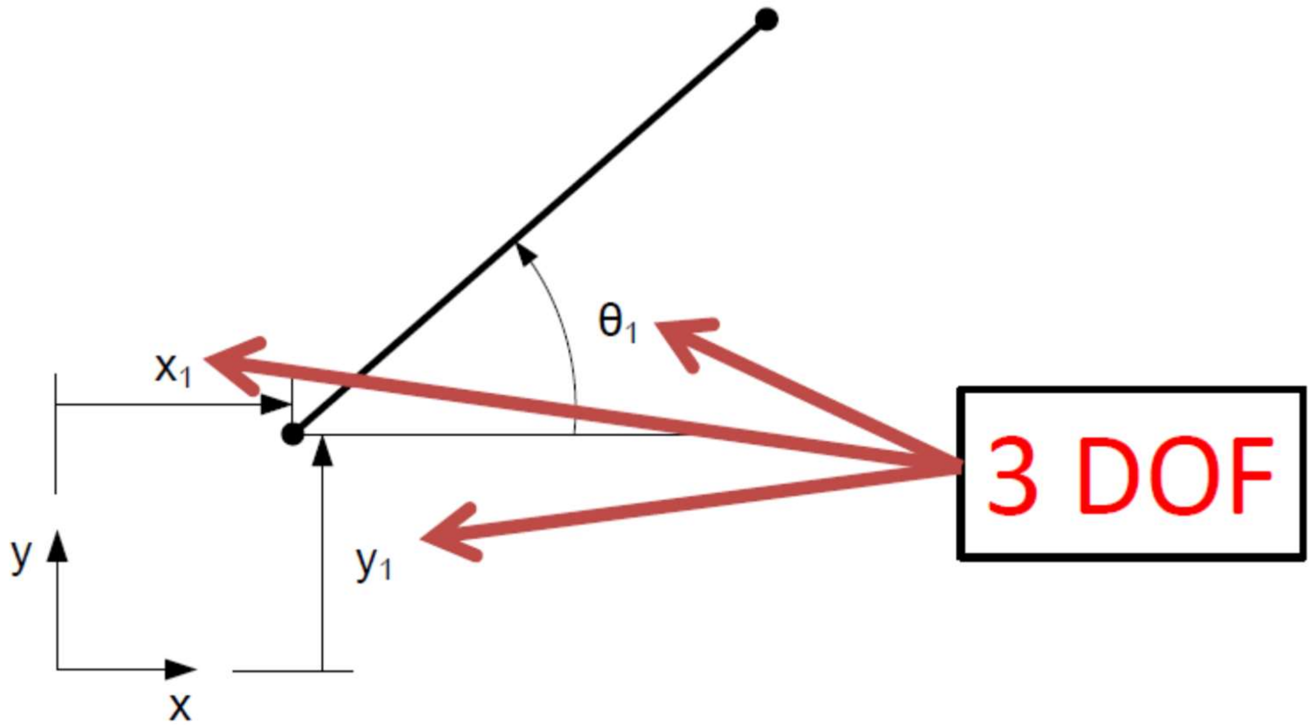


For example, if each of the above links are restricted to move in a plane (as part of a planar mechanism), then they would each have 3 dof (translation in x and y, and rotation θ).



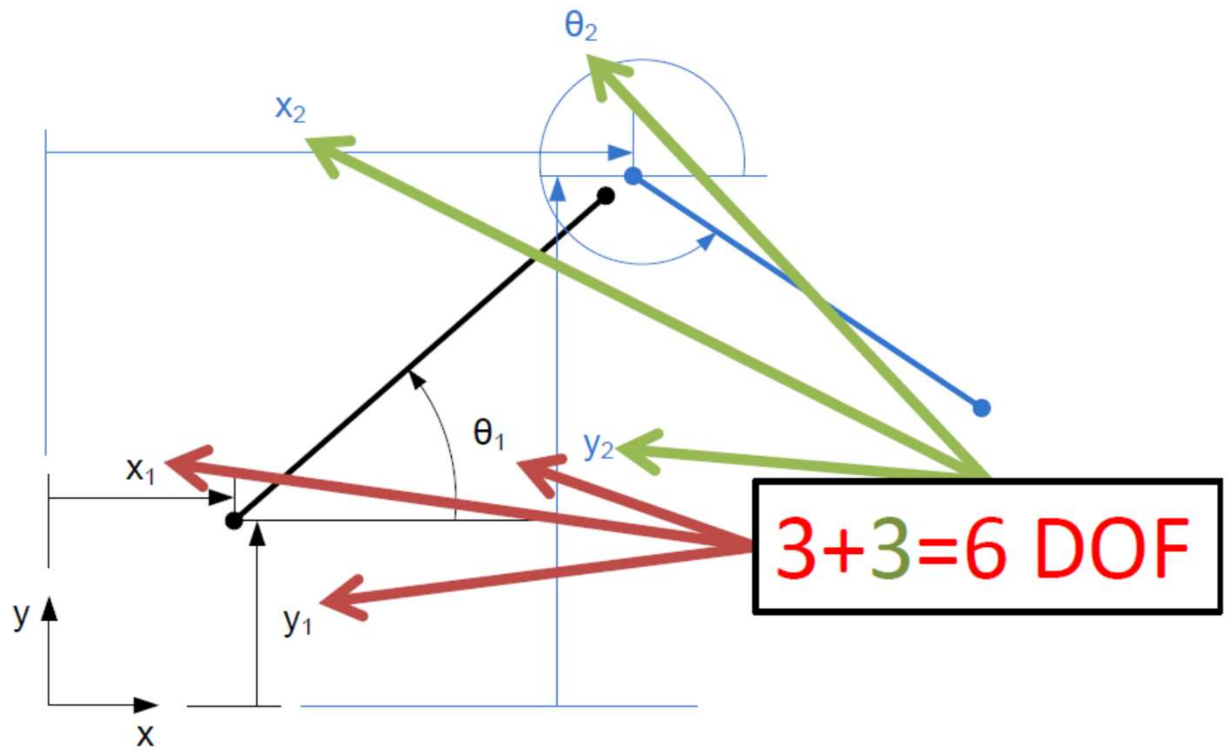
Degrees of Freedom (DOFs)

EXAMPLE: Consider a single link in the plane



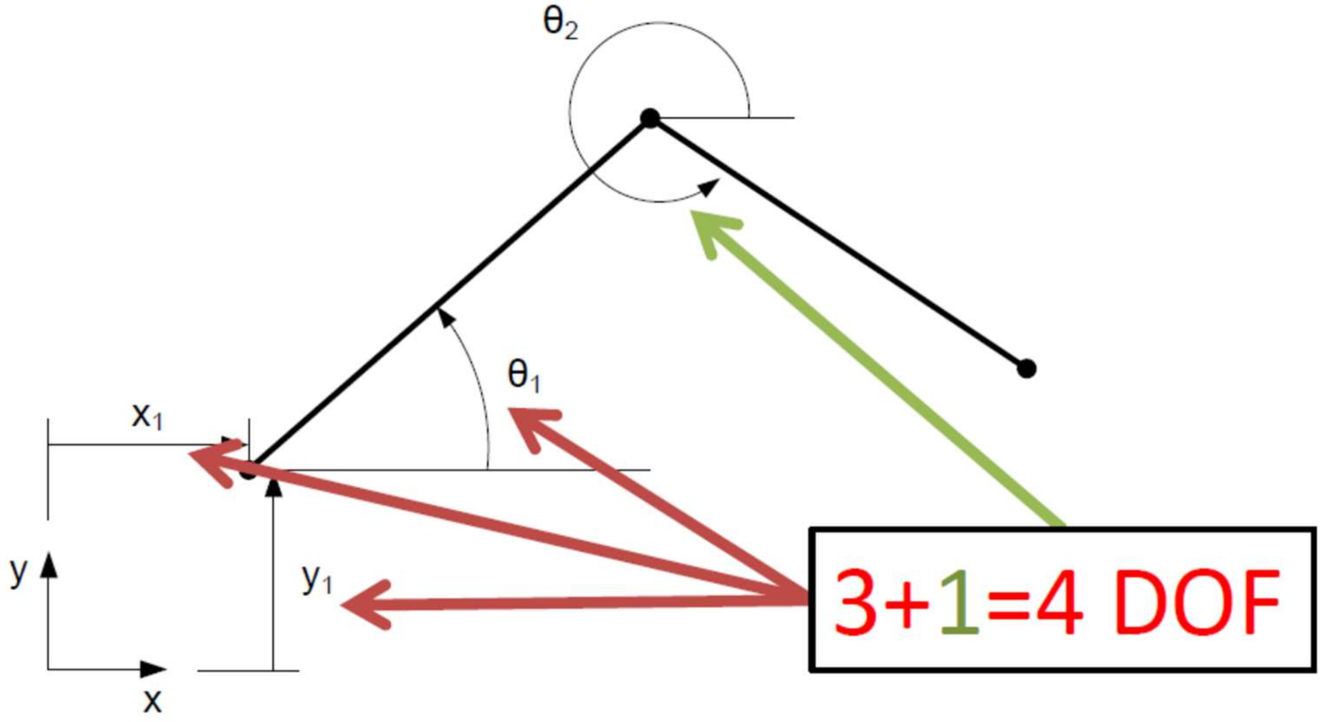
Degrees of Freedom (DOFs)

Adding another free link adds another 3 DOF



Degrees of Freedom (DOFs)

But joining the two links at a revolute joint reduces the total DOF by 2:



Calculation of Degree of Freedom (DoF)

- **Degree of Freedom (DoF):** the number of inputs that need to be provided to create a predictable output

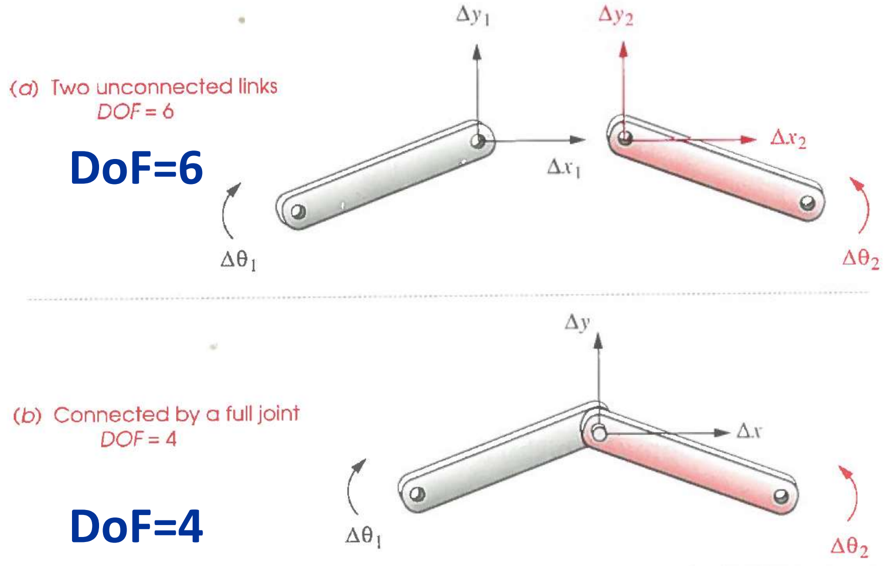
- **Gruebler's equation of DoF**:

$$M = 3L - 2J - 3G$$

where, M = degree of freedom
L = number of links
J = number of joints
G = number of grounded links

As there is only one grounded link,

$$M = 3(L-1) - 2J$$



Calculation of Degree of Freedom (DoF)

- **Gruebler's equation of DoF**:

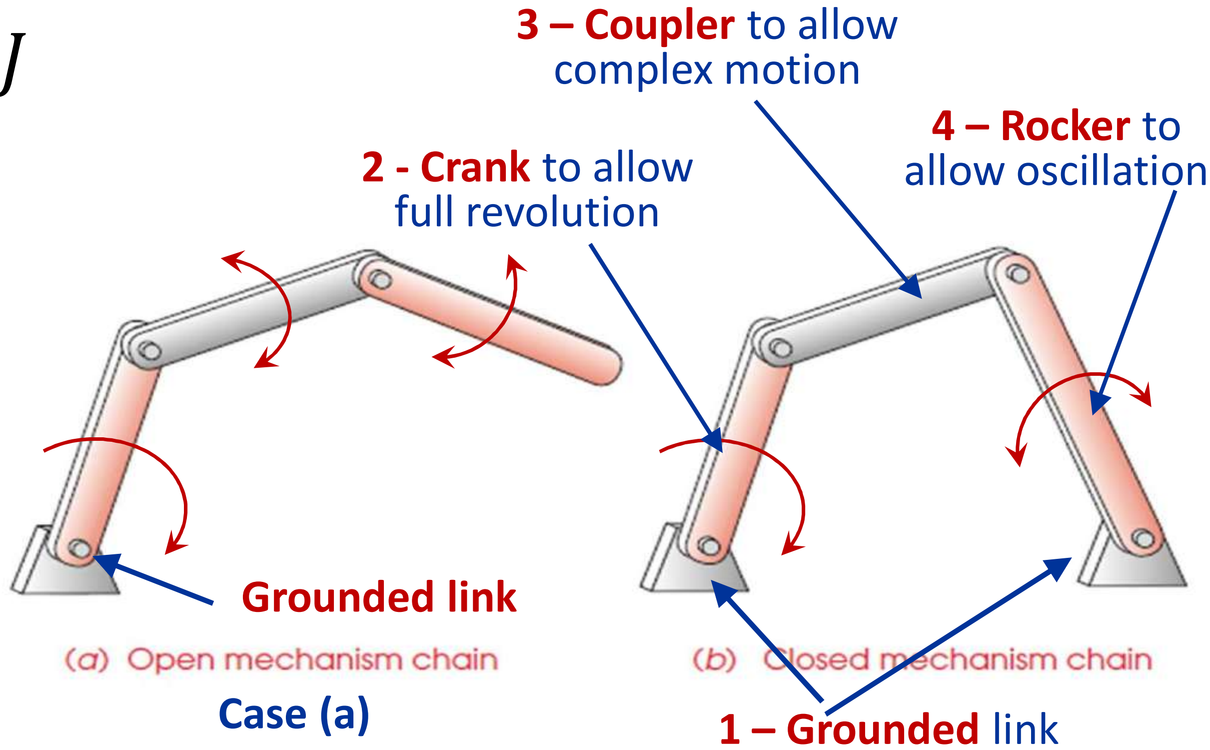
$$M = 3(L - 1) - 2J$$

- Case (a): L=4, J=3

  M = 3*(4-1) - 2*3 = **3**

- Case (b): L=4, J=4

  M = 3*(4-1) – 2*4 = **1**



Case (b) A typical 4-bar (crank-rocker) linkage mechanism

Two more examples of DoF calculation

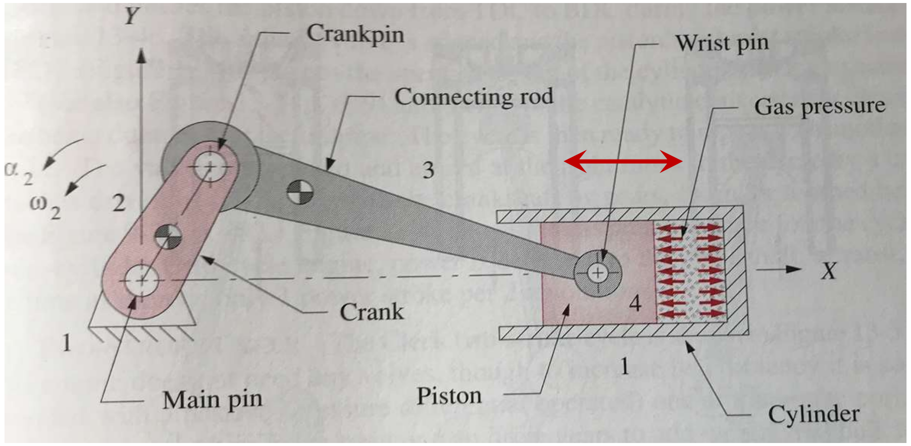
- **Gruebler's equation of DoF:**

$$M = 3(L - 1) - 2J$$

Slider is considered a **link**, L=4,

Piston sliding as a joint, J=4

M = 3*(4-1) – 2*4 = 1

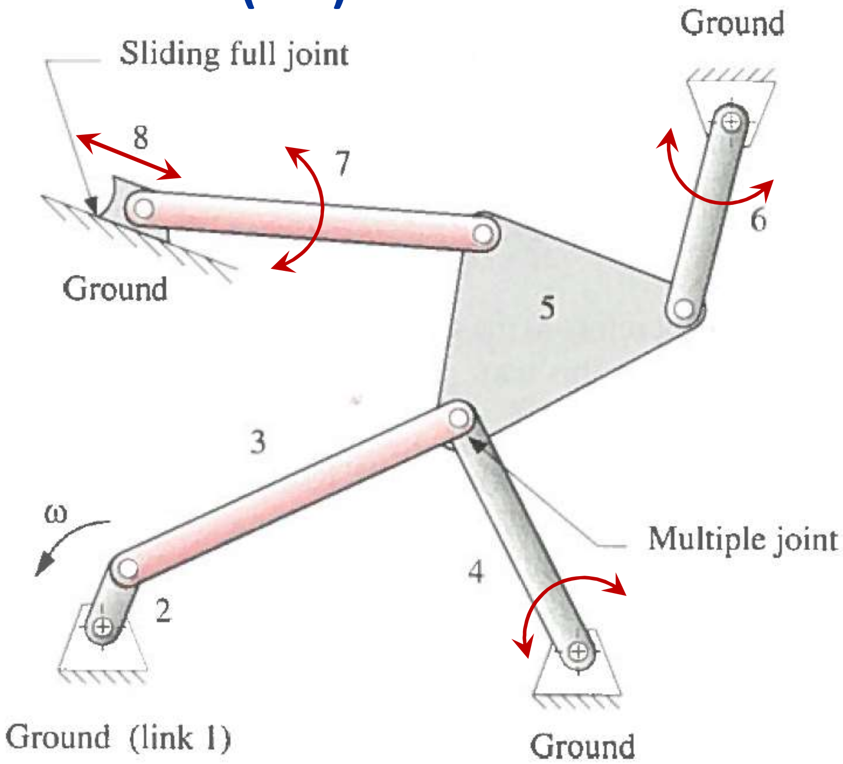


A slider-crank mechanism for a small IC engine

Slider is considered a **link**, L=8,

3 links is counted as 2 joints, J=10

M = 3*(8-1) – 2*10 = 1



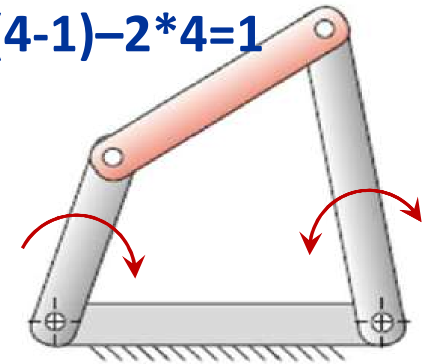
A linkage with multiple joints

Mechanisms and structures

- **DoF** of an assembly of links completely defines its **characteristics** with **THREE possibilities**.
  - If DOF is **positive (>0)**, it is a mechanism with relative motions
  - If DOF is **equal to zero (=0)**, it is a structure without motion
  - If DOF is **negative (<0),** it is a preloaded (or overly constrained) structure without motion but possibly additional stresses

L=4, J=4

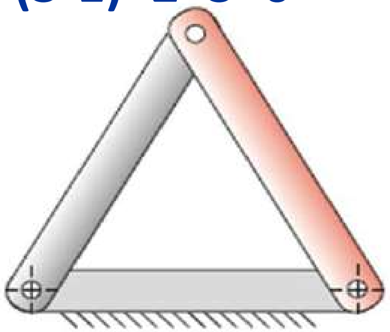
M=3*(4-1)–2*4=1



(a) Mechanism—*DOF* = +1

L=3, J=3

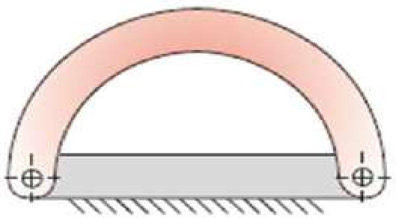
M=3*(3-1)–2*3=0



(b) Structure—*DOF* = 0

L=2, J=2

M=3*(2-1)–2*2= -1



(c) Preloaded structure—*DOF* = −1

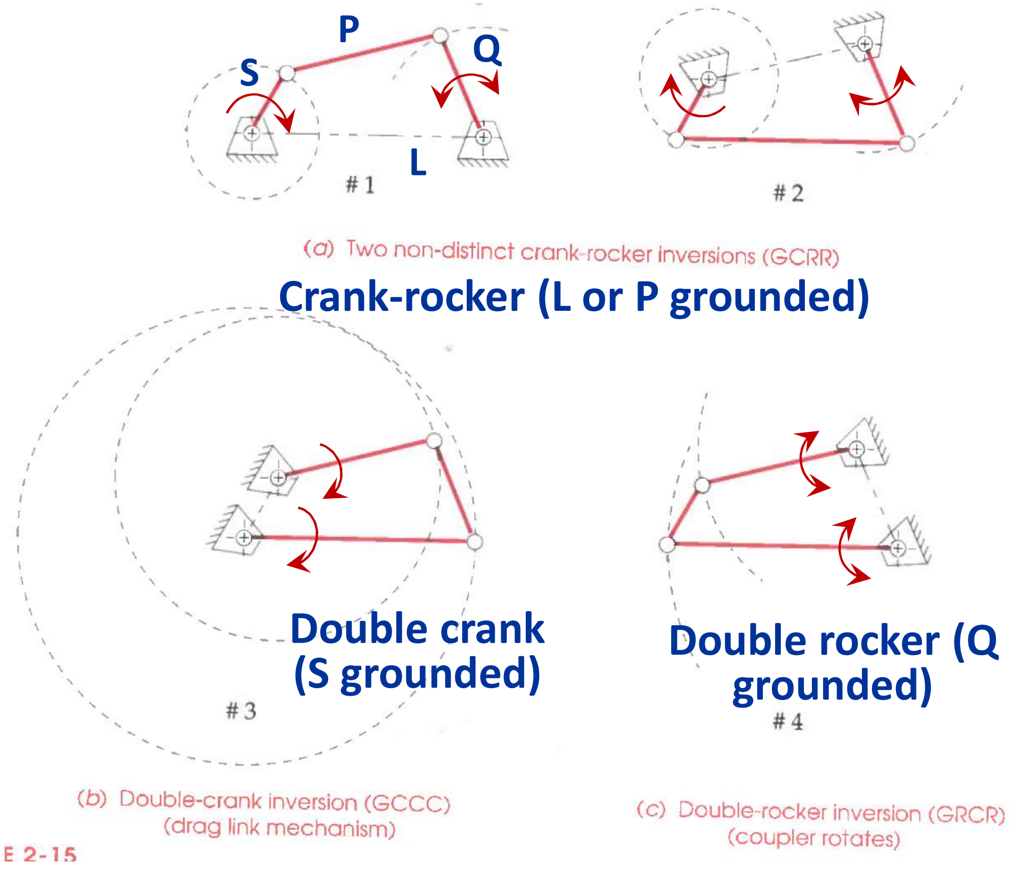
Grashof condition

- **Four bar linkage** is the simplest possible **pin-jointed** mechanism, versatile, cost-effective and reliable for many applications.
- **Grashof condition**: a simple relationship that predicts the linkage behaviour based only on **link lengths**

Let **S** = length of shortest link
L = length of longest link
P = length of one remaining link
Q = length of other remaining link

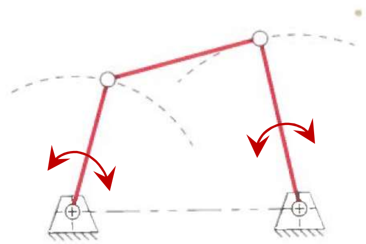
- **If S + L < P +Q**, a **Grashof linkage** with at least one link capable of making a full revolution

https://www.youtube.com/watch?v=h8bz4ni6mdY

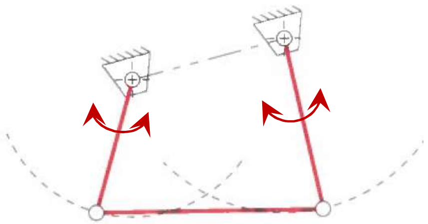


Grashof condition

- **If S + L > P +Q**, a **non-Grashof linkage** with **no link** capable of making a full revolution **(triple-rockers)**
- **If S + L = P +Q**, a **special case Grashof linkage,** either **double-cranks** or **crank-rockers** with change points

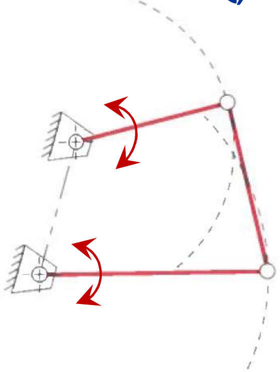


(a) Triple-rocker #1 (RRR1)

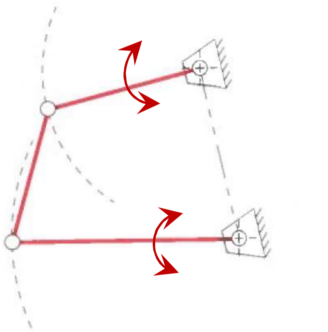


(b) Triple-rocker #2 (RRR2)

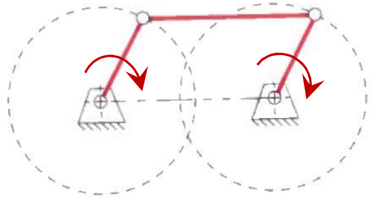
S + L > P +Q, non-Grashof (triple rocker)



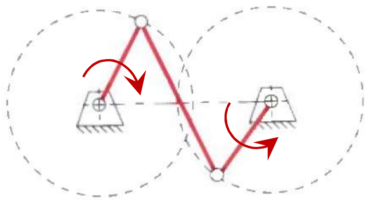
(c) Triple-rocker #3 (RRR3)



(d) Triple-rocker #4 (RRR4)

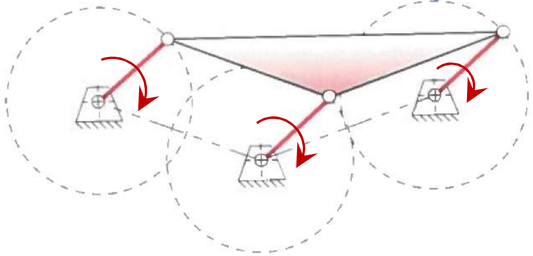


(a) Parallelogram form

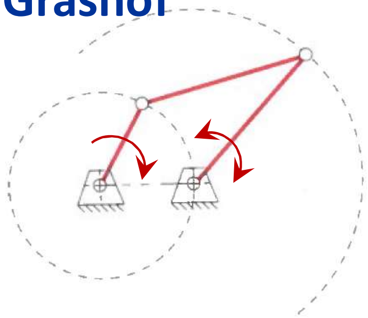


(b) Antiparallelogram form

S + L = P +Q, special-case Grashof



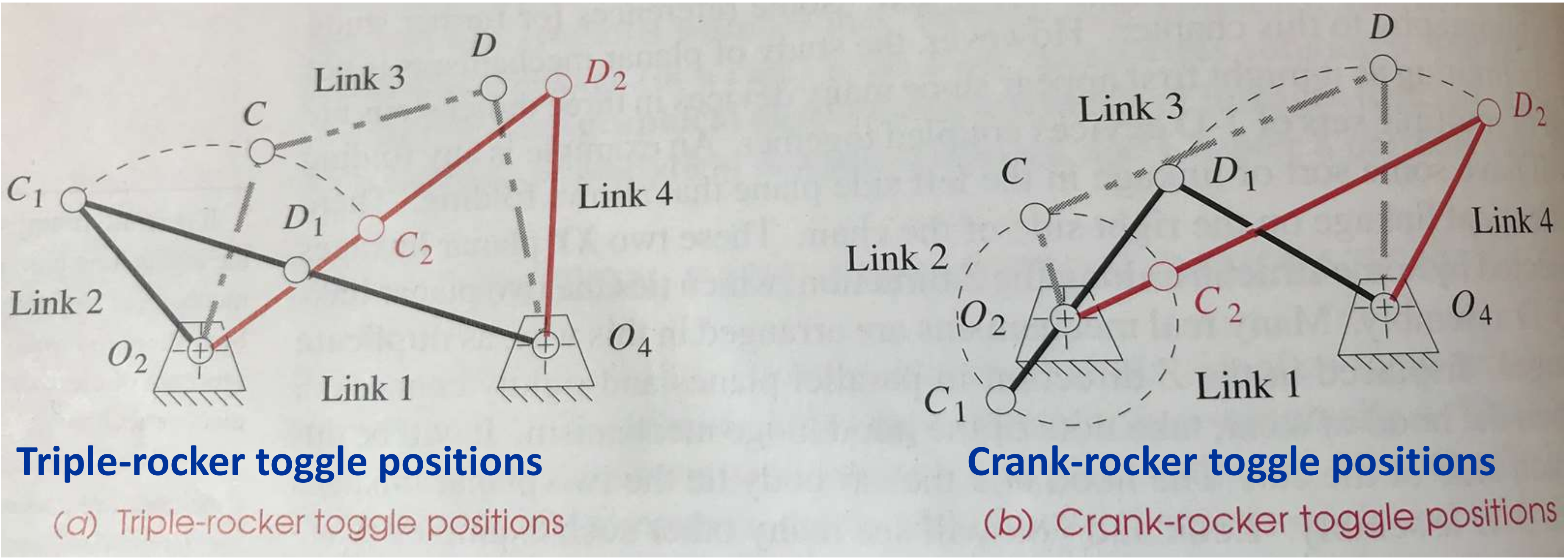
(c) Double-parallelogram linkage gives parallel motion (pure curvilinear translation) to coupler and also carries through the change points



(d) Deltoid or kite form

Limit conditions

- **Toggle** is an **important test** to check if the linkage can reach all specified positions without encountering a **limit** or **toggle** position, also called a stationary configuration.
- Toggle positions may be determined by **collinearity** of two links.



Triple-rocker toggle positions

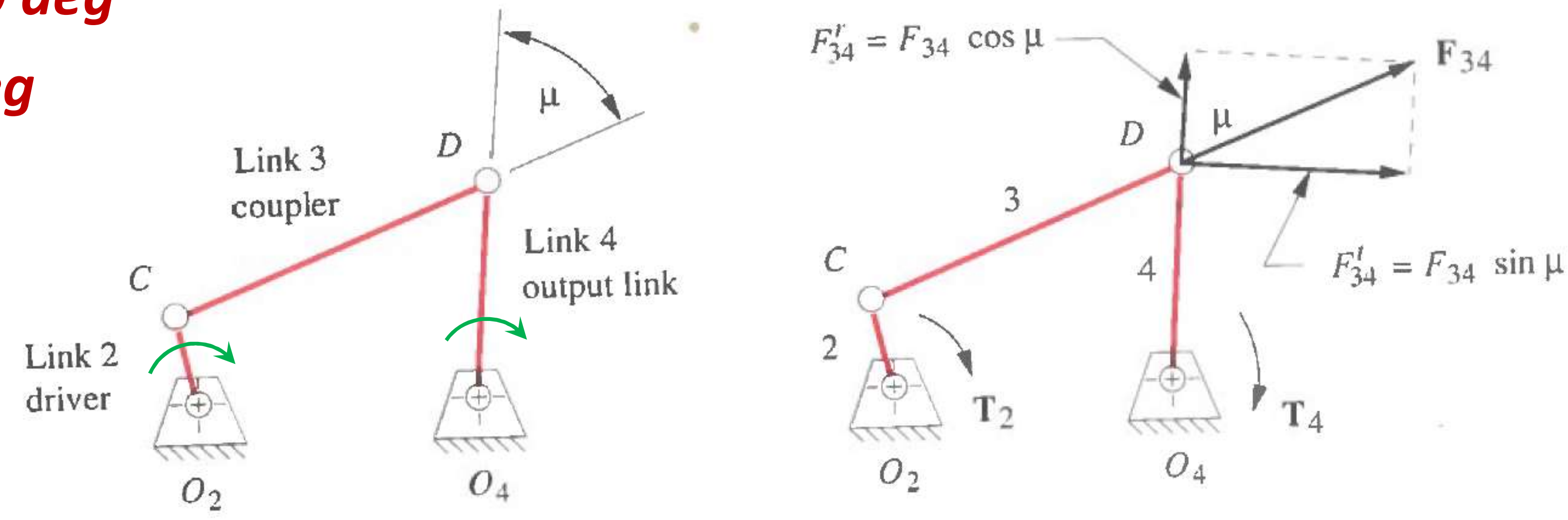
Crank-rocker toggle positions

Transmission angle

- **Transmission angle**, ***μ***, is the angle between the **output link** and the **coupler**
- **Transmission angle, *μ***, is another useful test to check the quality of force transmission in a linkage

Ideally, *μ = 90 deg*

Min μ ≥ 40 deg



(a) Linkage transmission angle μ

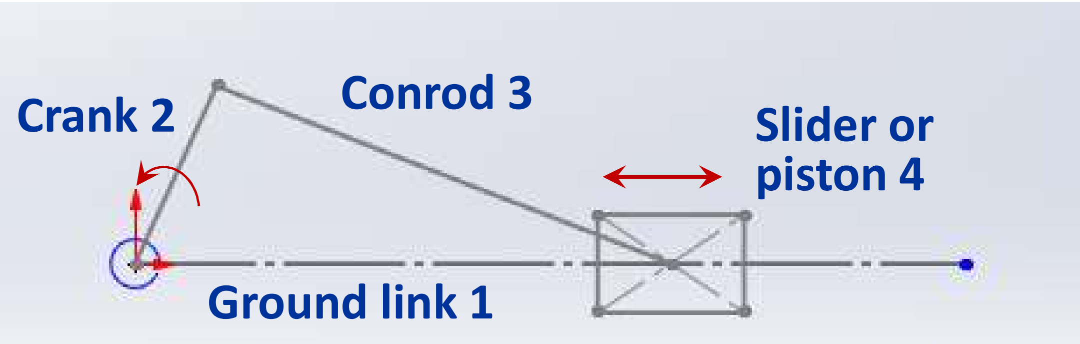
(b) Static forces at a linkage joint

Validation of Gruebler's DoF equation

- **Gruebler's equation of DoF**: (slide 10)

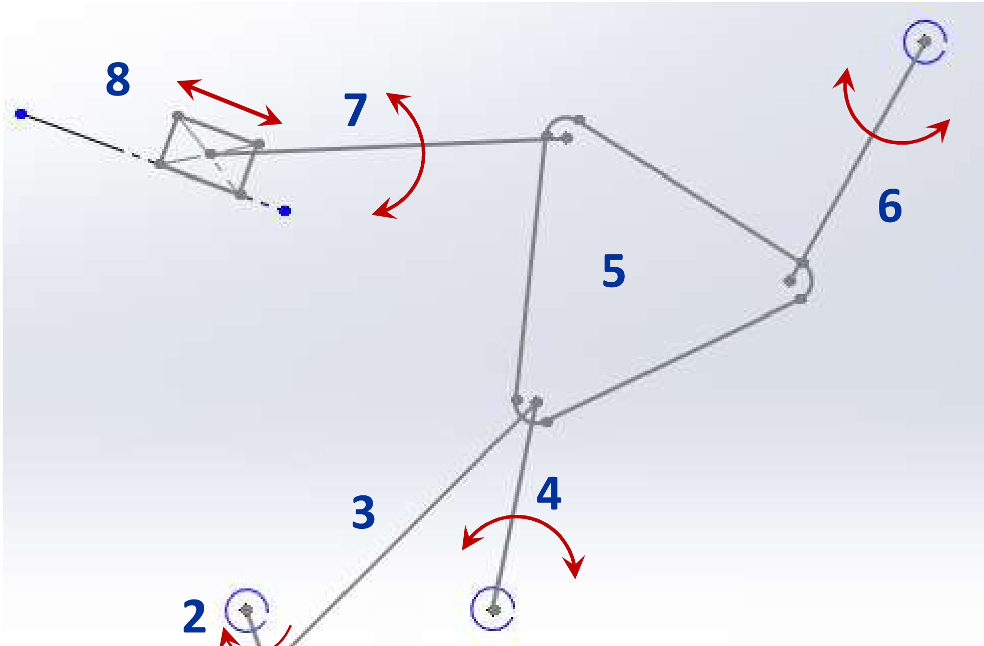
$$M = 3(L - 1) - 2J$$

Slider is considered a **link**, L=4,
Piston sliding as a joint, J=4
M = 3*(4-1) – 2*4 = 1



A slider-crank mechanism

Slider is considered a **link**, L=8,
3 links is counted as 2 joints, J=10
M = 3*(8-1) – 2*10 = 1



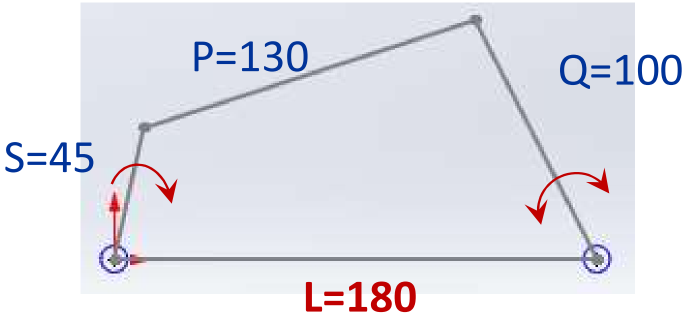
A linkage with multiple joints

Grashof and non-Grashof 4-bar linkages

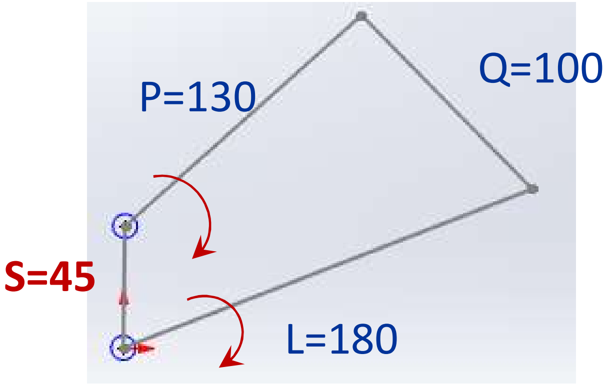
(slides 12~13)

- **Grashof linkages (S + L < P +Q)**

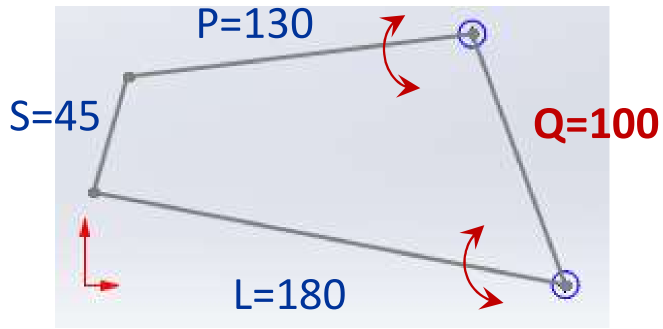
S = shortest link; **L** = longest link;
P = one remaining link; **Q** = other remaining link



Crank-rocker (L grounded)

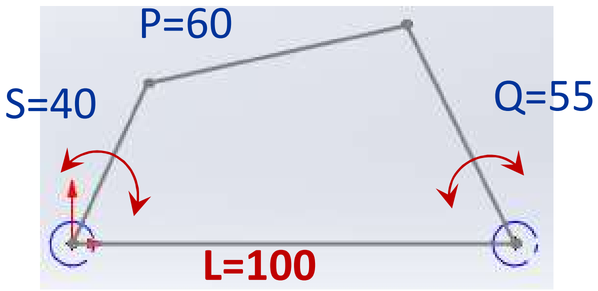


Double crank (S grounded)



Double rocker (Q grounded)

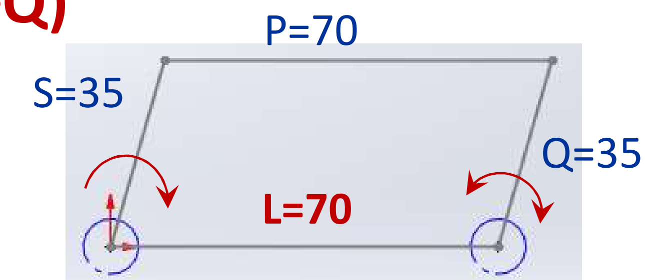
- **Non-Grashof linkage (S + L > P +Q)**



non-Grashof (triple rocker)

Question: How to make a Parallelogram linkage

- **A special case of Grashof linkage (S + L = P +Q)**

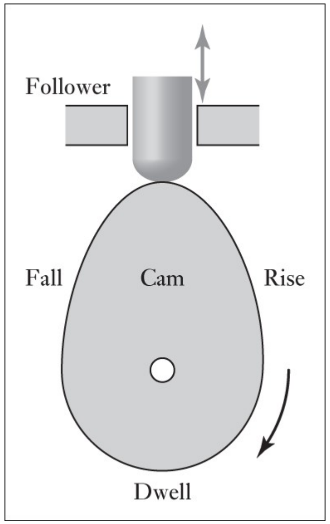


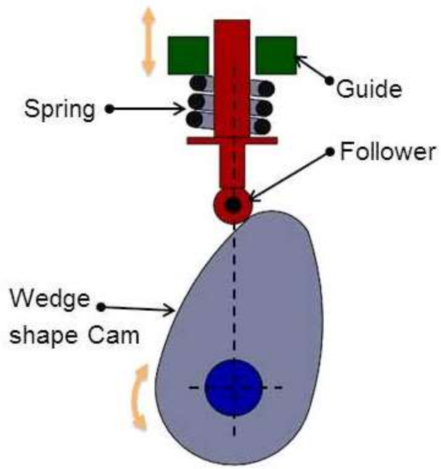
Special case of Grashof (crank rocker with change point)

Cams

- As the cam rotates so the follower is made to rise, dwell and fall; the length of times spent at each of these positions depends on the shape of the cam.

  - **The rise section** is the one that drives the follower upwards.
  - **The fall section** is the one that lowers the follower.
  - **The dwell section** is the one that allows the follower to remain at the same level for a significant period of time. It is circular with a radius that does not change





Cams

- While the **cam rotates**, the follower is moving with a displacement as shown in the diagram.

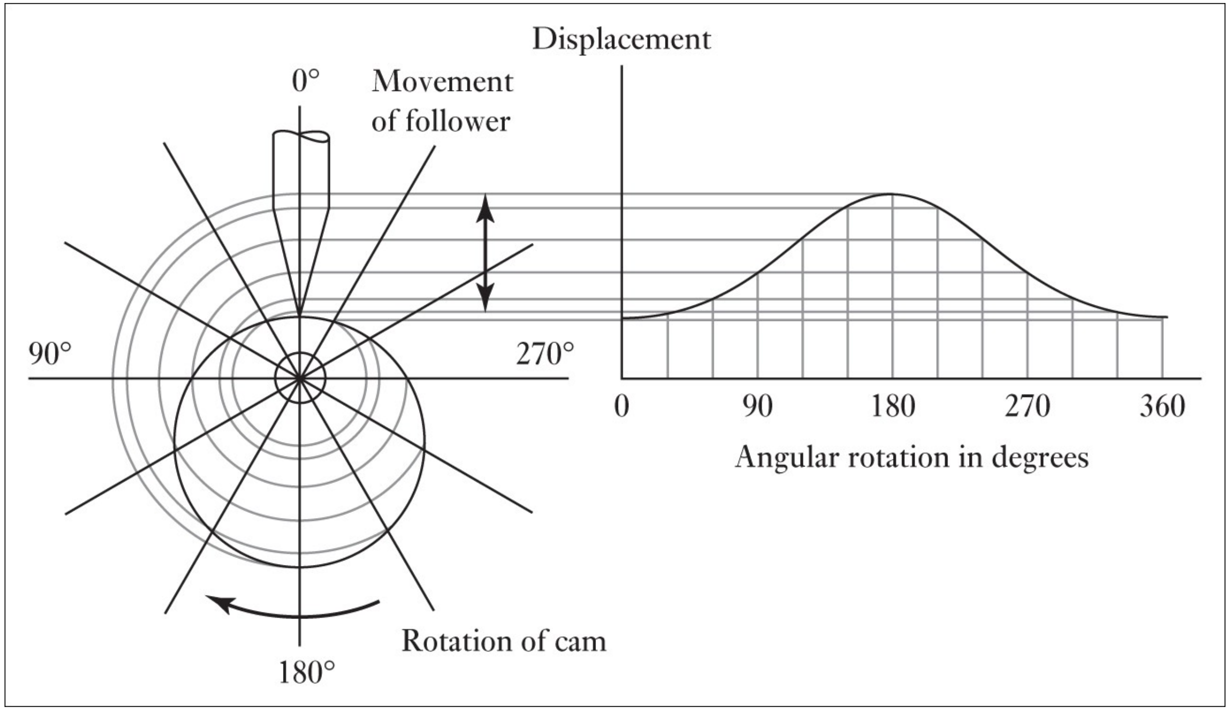


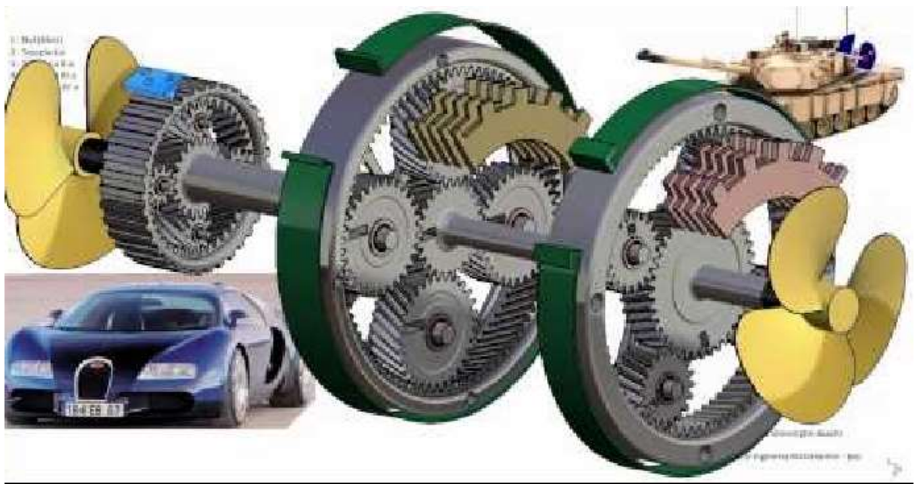
Figure Displacement diagram for an eccentric cam.

Gears and Gear Trains

- Are used to transfer and transform rotational motion.

- They are used when a change in speed or torque of a rotating device is needed.

  e.g. the car gear box enables the driver to match the speed and torque requirements of the terrain with the engine power available.



Gears and Gear Trains

Parallel shaft gears:

- Spur gears: have axial teeth with the teeth cut along axial lines parallel to the axis of the shaft
- Helical gears: helical teeth with teeth being cut on helix helical gears have the advantage of smoother drive and prolonged life of gears, however, the inclination of the teeth results in an axial force component on the shaft bearing which can be overcome by using double helical teeth.
- Rack and Pinion: are used to transfer linear to rotary motion.

Spur gears

Helical gears

Rack and Pinion

Gears and Gear Trains

Gears with intersecting shafts

- Rotary motion can be transferred from one shaft to another by a pair of rolling cylinders.

- The transfer of the motion between the two cylinders depends on the frictional forces between the two surfaces in contact . Slip can be prevented by the addition of meshing teeth to the two cylinders

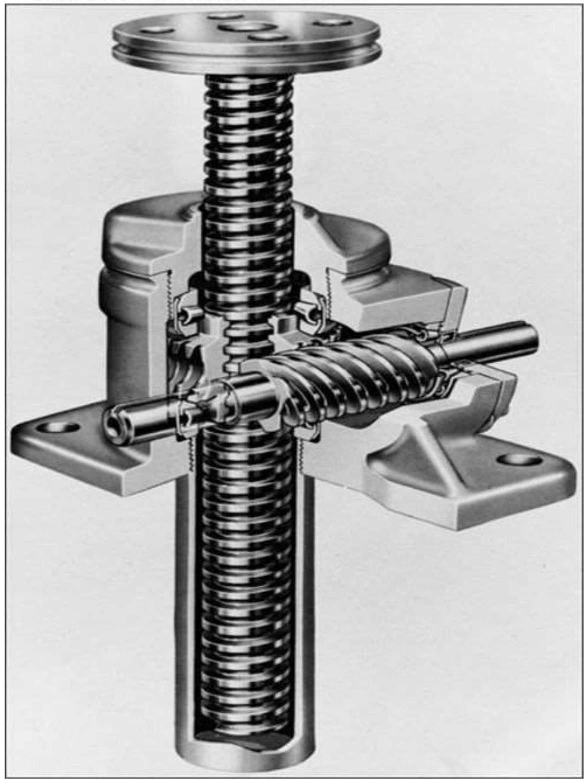
- Also gears can be used to transfer rotating motion for shafts which have axis inclined to one another, i.e. the two shafts intersect (bevel gears).

Bevel gears

Gears and Gear Trains

Power Screw

- A power screw is a device that is common to tools or machinery that are used to change angular motion into translation. It is also capable of developing a large amount of mechanical advantage. Familiar applications include clamps or vises, presses, lathes lead screws, and jacks.



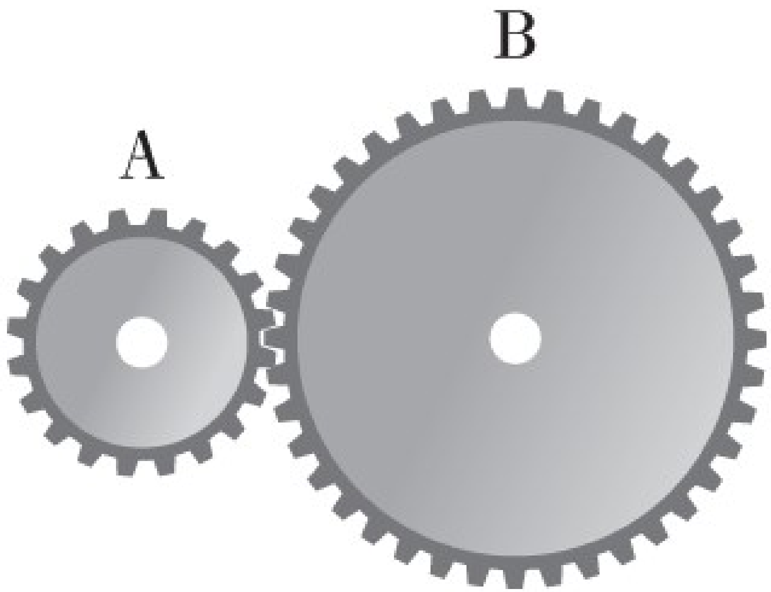
Gears and Gear Trains

Consider meshed wheels A and B. A with 40 teeth and B with 80 teeth.

$$\frac{\omega_A}{\omega_B} = \frac{No\ of\ teeth\ on\ B}{No\ of\ teeth\ on\ A} = \frac{80}{40} = 2$$

$$\omega_A = 2\omega_B$$

$$\frac{\omega_A}{\omega_B} = \frac{d_B}{d_A} = gear\ ratio = 2$$



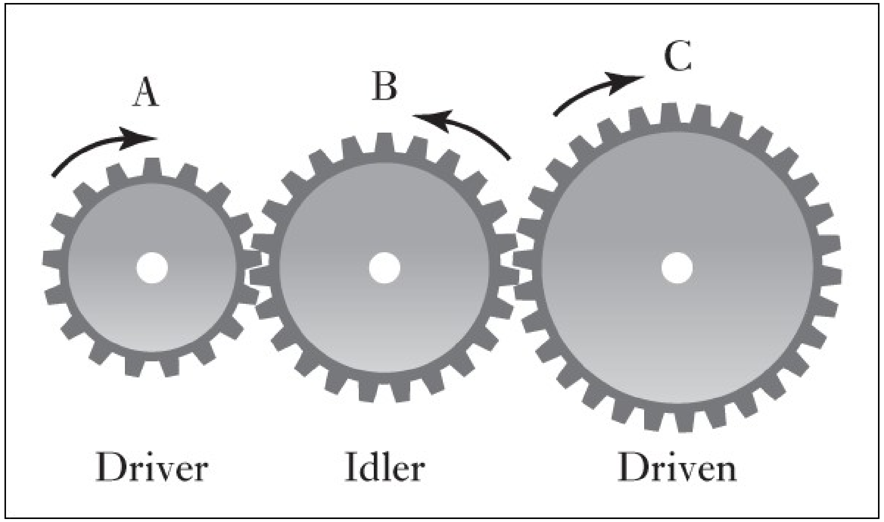
- Wheel B must have twice the diameter of wheel A.
- Gear ratio is used for the ratio of the angular speeds of a pair of intermeshed gear wheels

Gears and Gear Trains

Simple Gear Train

Each shaft carries only one gear wheel.

$$G = \frac{\omega_A}{\omega_C} = \frac{\omega_A}{\omega_B} \times \frac{\omega_B}{\omega_C}$$

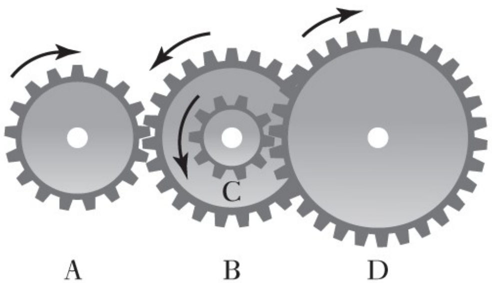


- The intermediate wheel B is termed the idler wheel, is used to change the direction of rotation of the output wheel.

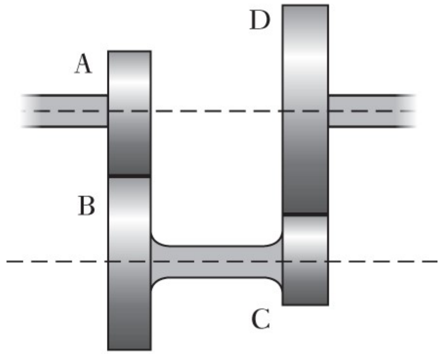
Gears and Gear Trains

Compound Gear Train

This term is used to describe a gear train when two (or more) wheels are mounted on a common shaft. When two wheels are mounted on the same shaft, they have the same angular velocity.



(a)



(b)

$$\omega_B = \omega_C$$

The over all gear ratio:

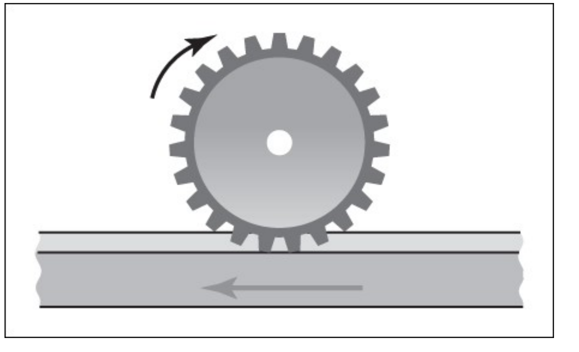
$$G = \frac{\omega_A}{\omega_D} = \frac{\omega_A}{\omega_B} \times \frac{\omega_B}{\omega_C} \times \frac{\omega_C}{\omega_D} = \frac{\omega_A}{\omega_B} \times \frac{\omega_C}{\omega_D}$$

Rotational to Translational Motion

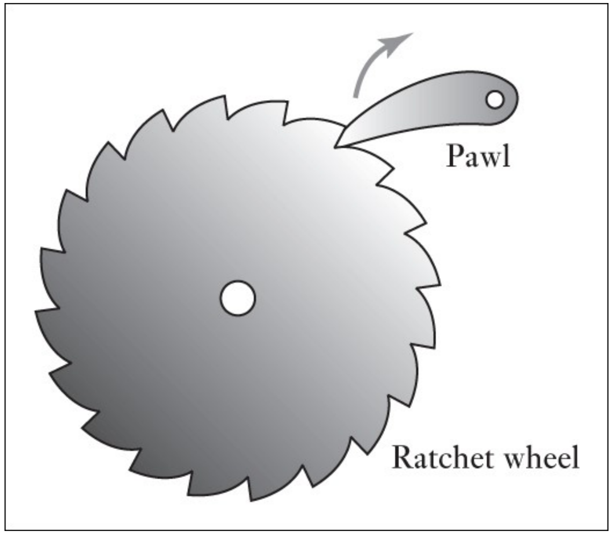
- A rack and pinion is a type of linear actuator that comprises a pair of gears which convert rotational motion into linear motion.

- A circular gear called "the pinion" engages teeth on a linear "gear" bar called "the rack";

- Rotational motion applied to the pinion causes the rack to move relative to the pinion, thereby **translating the rotational motion of the pinion into linear motion**.

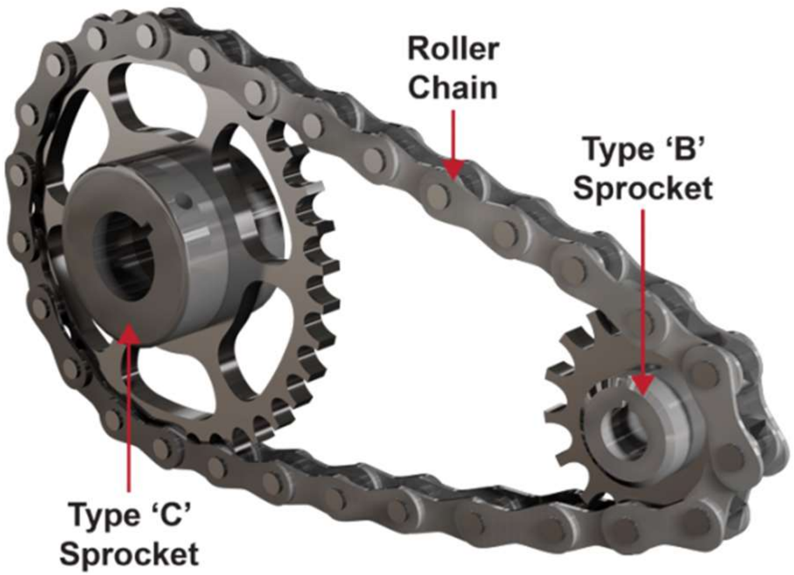


Racket and Pawl



It consist of a wheel, called a ratchet, with saw-shaped teeth which engage with an arm called Pawl. The arm is pivoted and can move back and forth to engage the wheel. The shape of the teeth is such that rotation can occur in only one direction. (used in winch to wind up a cable on drum)

Belt and Chain Drives



- A pair of rolling cylinders with the motion of one cylinder being transferred to the other by a belt.

Belt and Chain Drives

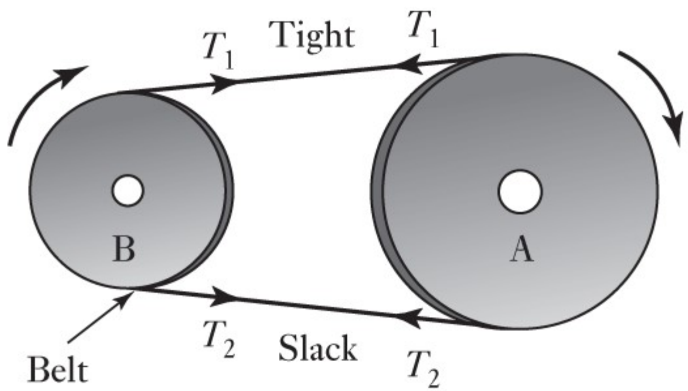
- The transmitted torque is due to differences in tension that occur in the belt during operation. This difference results in a tight side and slack side for the belt.

- If the tension on the tight side is T1 and that on the slack side is T2 then:

Torque on A =(T1-T2)r_A

Torque on B=(T1-T2)r_B

r_A and r_B are the radius of A and B



If belt speed is v then angular speed for a and B are:

$$\omega_A = \frac{v}{r_A} \quad \& \quad \omega_B = \frac{v}{r_B}$$

Power on either pulley is: $P = (T1 - T2)v$

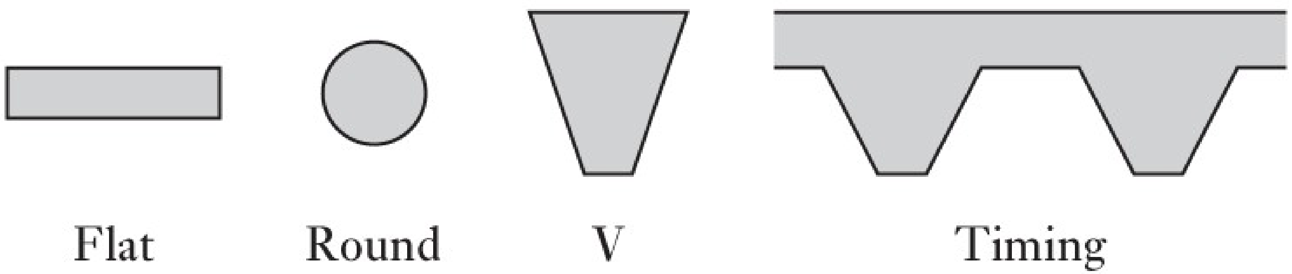
Belt and Chain Drives: Types of Belts

- **Flat:** produces little noise, can transmit power over long distance. Crowned pulleys are used to keep the belts from running off the pulleys

- **Round**: has circular cross section and is used with grooved pulleys

- **V belts**: are used with grooved pulleys, less efficient than flat belts

- **Timing belts**: required toothed wheels; it can run at slow or fast speed, does not stretch or slip



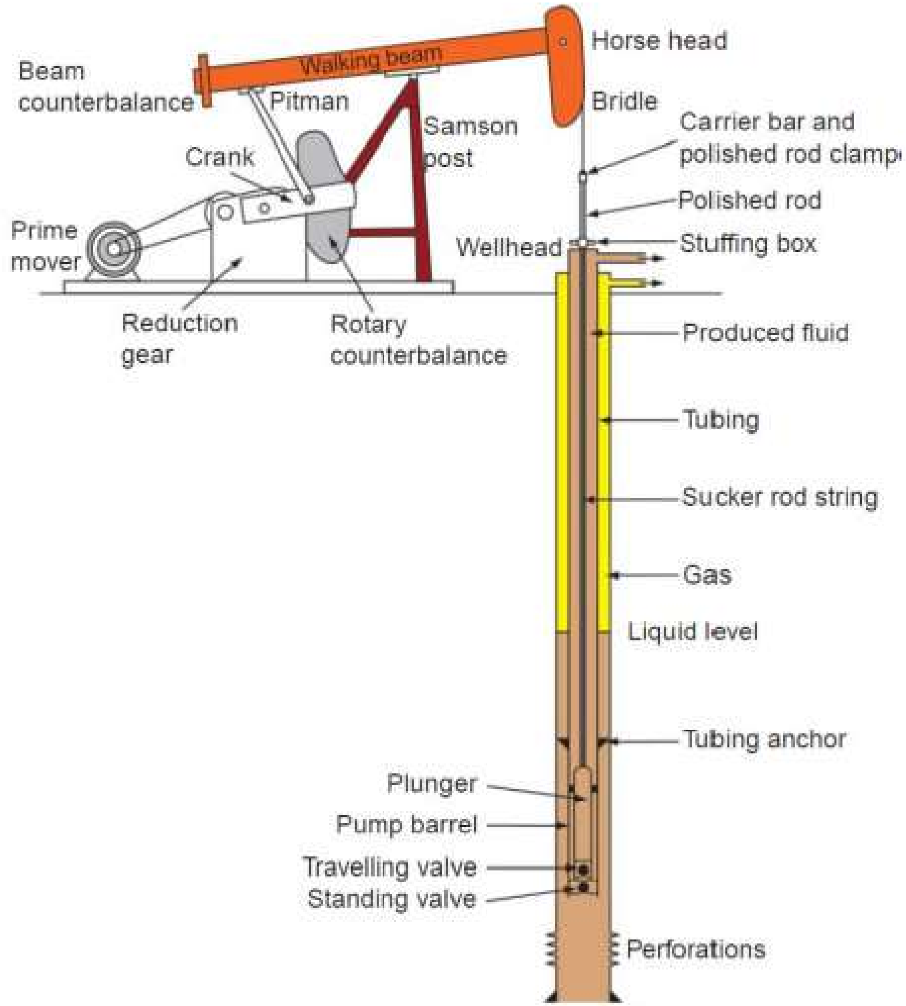
Chains

- Slip can be prevented by the use of chains which lock into teeth on the rotating cylinders.
- The drive mechanism used with bicycle is an example of a chain drive.
- It enables a number of shafts to be driven by single wheel and so give a multiple drive.

Various Linkage Mechanisms

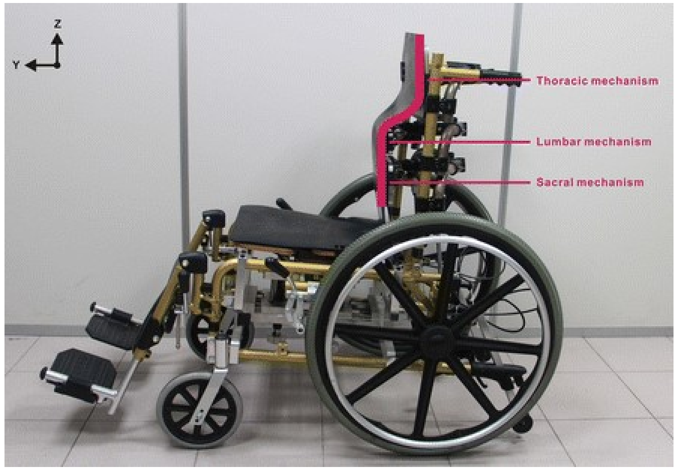
<u>Oil Field Pump (Sucker Pump)</u>

As the motor turns the crank (in orange), the walking beam (in yellow) oscillates. The pumping (sucker) rod, which is immersed in the oil, is connected to the horse-head of the walking beam by a cable. Therefore, the oscillation of the walking beam is converted to the reciprocating motion of the pumping rod to pump oil.



Various Linkage Mechanisms

Wheelchair Backrest for Elderly



Wheelchair with Sit to Stand (Video)

https://www.youtube.com/watch?v=v976i519Eng

Relaxed Slouching Sitting (RSS)

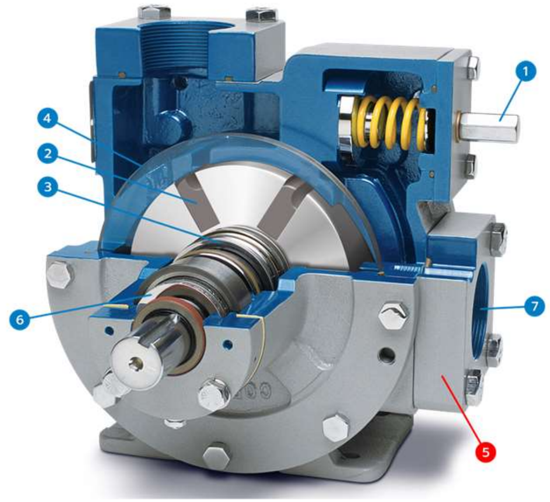
Flat Back Support Sitting (FBS)

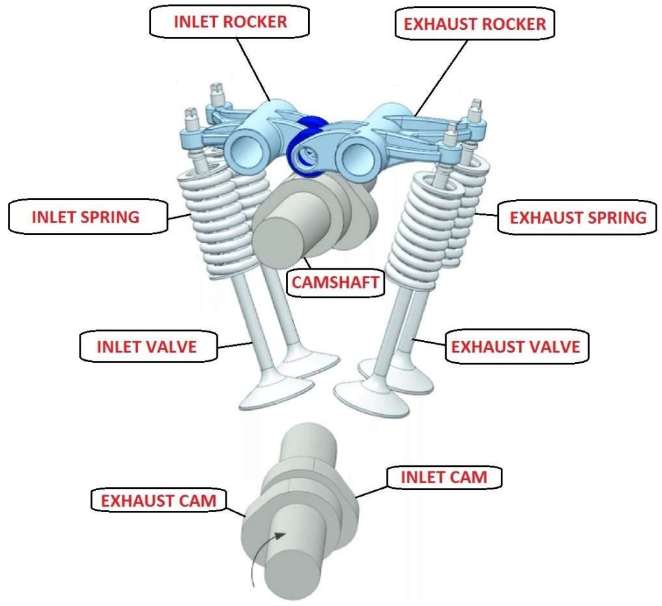
Prominent Lumbar Support Sitting (PLS)

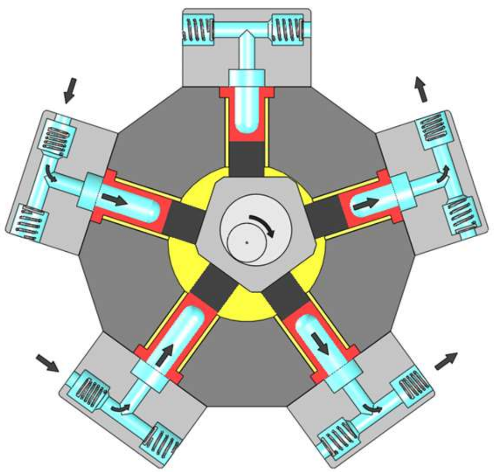
Backward Thoracic Support Sitting (BTS)

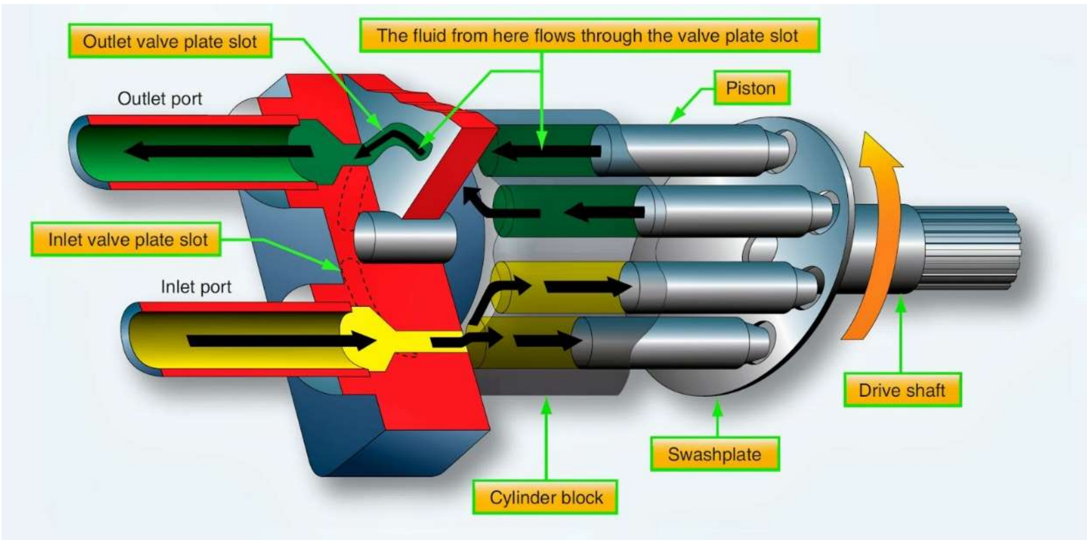
https://biomedical-engineering-online.biomedcentral.com/articles/10.1186/s12938-015-0008-6

Various Linkage Mechanisms







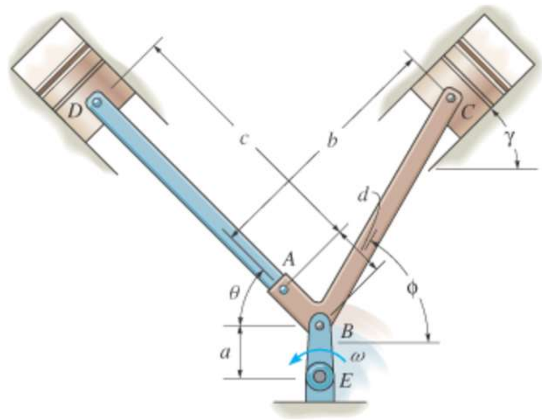


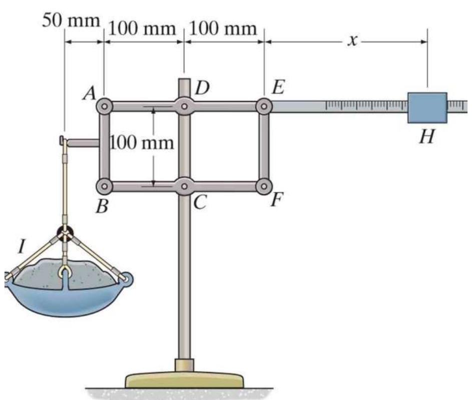
Name the mechanisms in all these figures!!

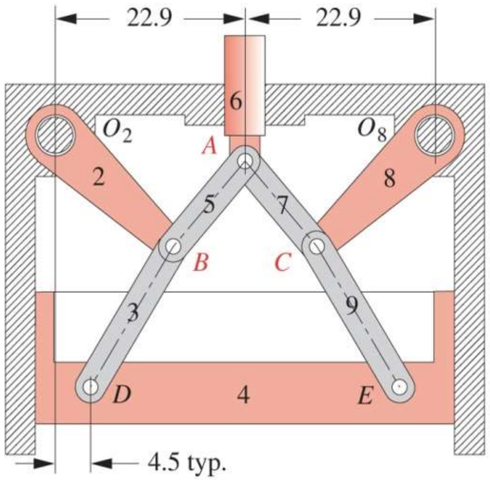
Revision Questions

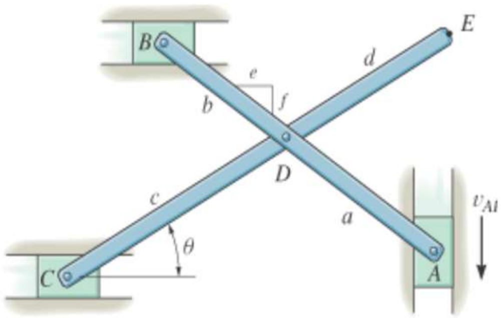
A. Identify DOFs of a system.
B. What is the gear ration in gear trains?
C. What are the differences between spur and helical gears?
D. Identify the three main actions in follower/cam motion.
E. Mention the four types of motion.
F. Mention 4 types of kinematic joints and state the DOFs for each type.

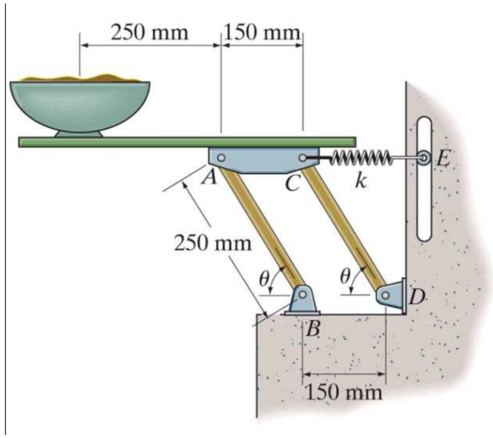
<u>Calculate the DOFs (Mobility) for all the five following linkages.</u>











Introduction to Linkage Mechanisms

End of Session